Dynamics of Prices in Electric Power Networks

Sean Meyn

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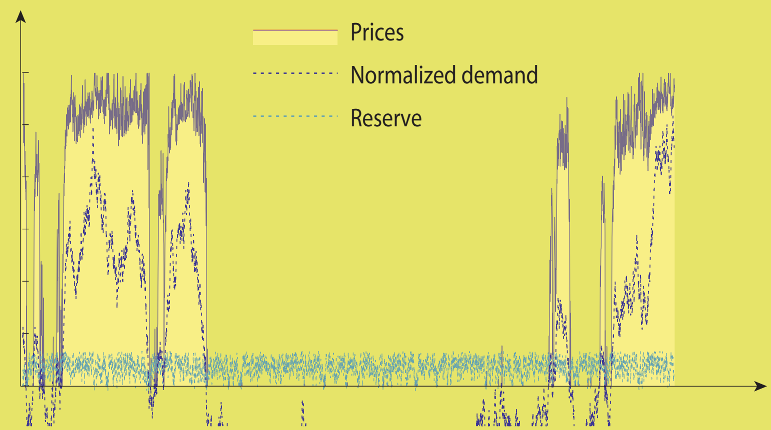
and the Coordinated Science Laboratory
University of Illinois

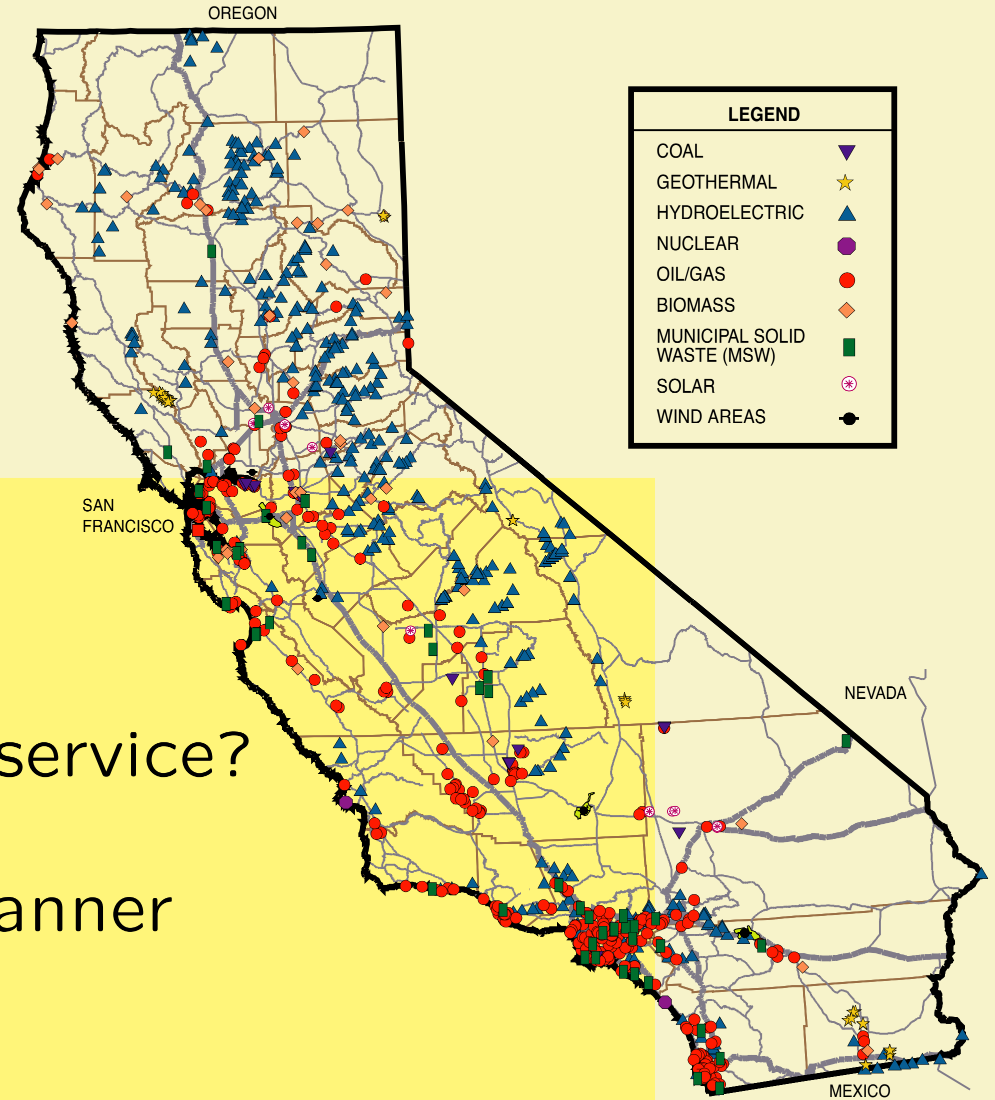


Joint work with M. Chen and I-K. Cho

NSF support: ECS 02-17836 & 05-23620 *Control Techniques for Complex Networks*

DOE Support: http://www.sc.doe.gov/grants/FAPN08-13.html
Extending the Realm of Optimization for Complex Systems: Uncertainty, Competition and Dynamics
PIs: Uday V. Shanbhag, Tamer Basar, Sean P. Meyn and Prashant G. Mehta

California's 25,000 Mile Electron Highway



What is the value
of improved transmission?
More responsive ancillary service?

How does a centralized planner
optimize capacity?

Is there an efficient decentralized solution?

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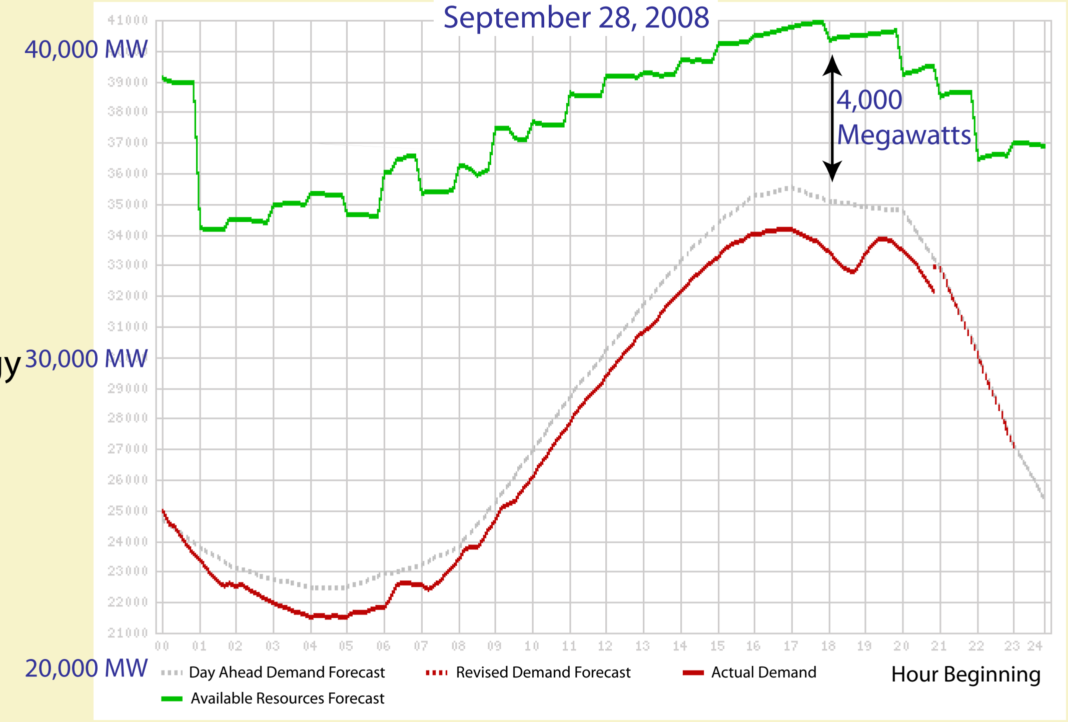
http://www.caiso.com/outlook/outlook.html



Available Resources

The current forecast of generating and import resources available to serve the demand for energy within the California ISO service area

Forecast Demand

Forecast of the demand expected today.

The procurement of energy resources for the day is based on this forecast

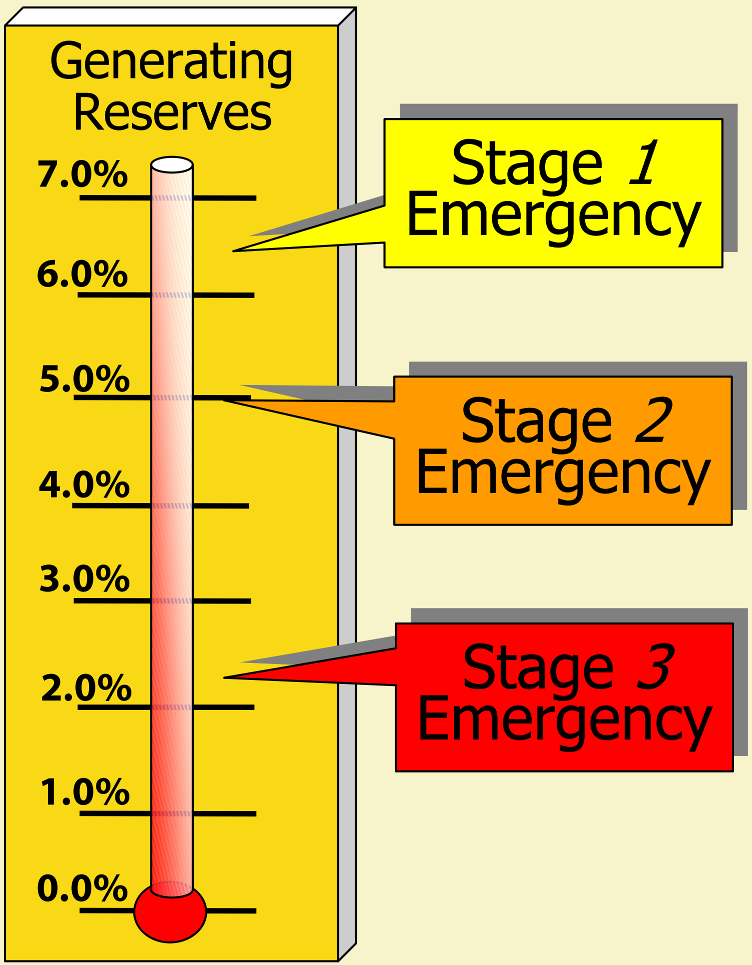
Actual Demand

Today's actual system demand

Revised Demand Forecast

The current forecast of the system demand expected throughout the remainder of the day. This forecast is updated hourly.

Emergency Notices



Generating reserves less than *requirements*

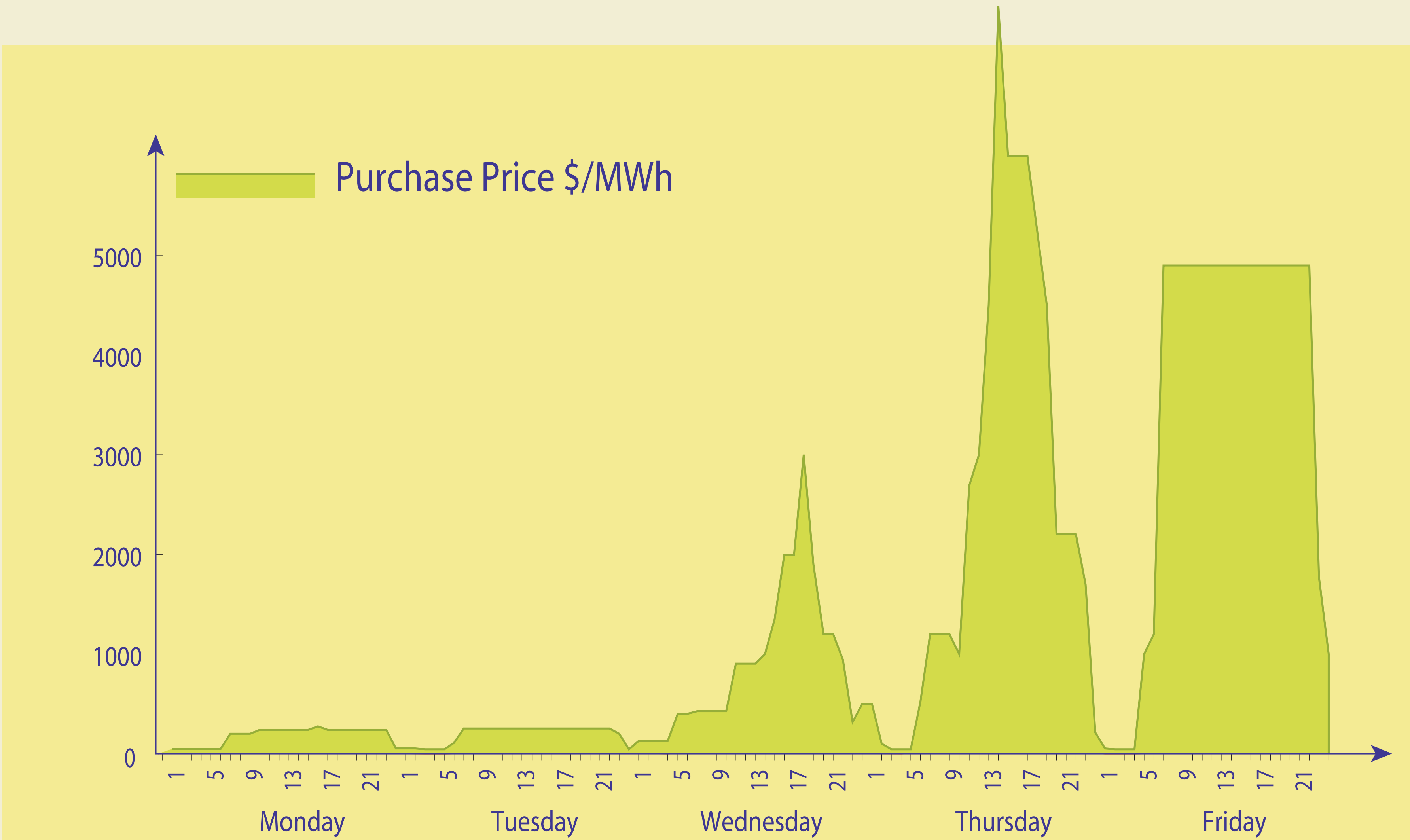
(Continuously recalculated. Between 6.0% & 7.0%)

Generating reserves less than *5.0%*

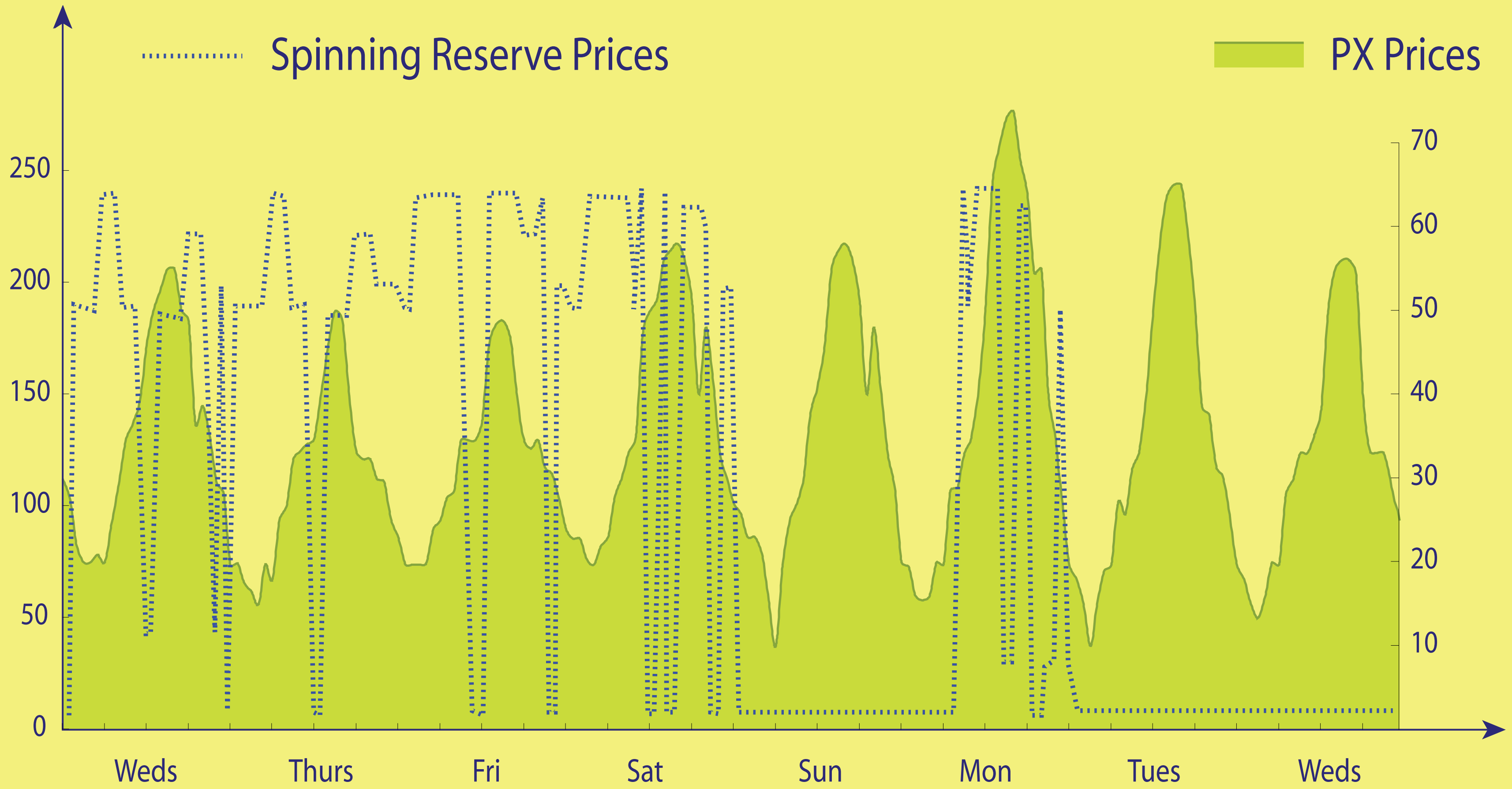
Generating reserves less than *largest contingency*

(Continuously recalculated. Between 1.5% & 3.0%)

One Hot Week in Urbana ...



Southern California, July 8-15, 1998 ...



First Impressions:

July 1998: first signs of "serious market dysfunction" in California

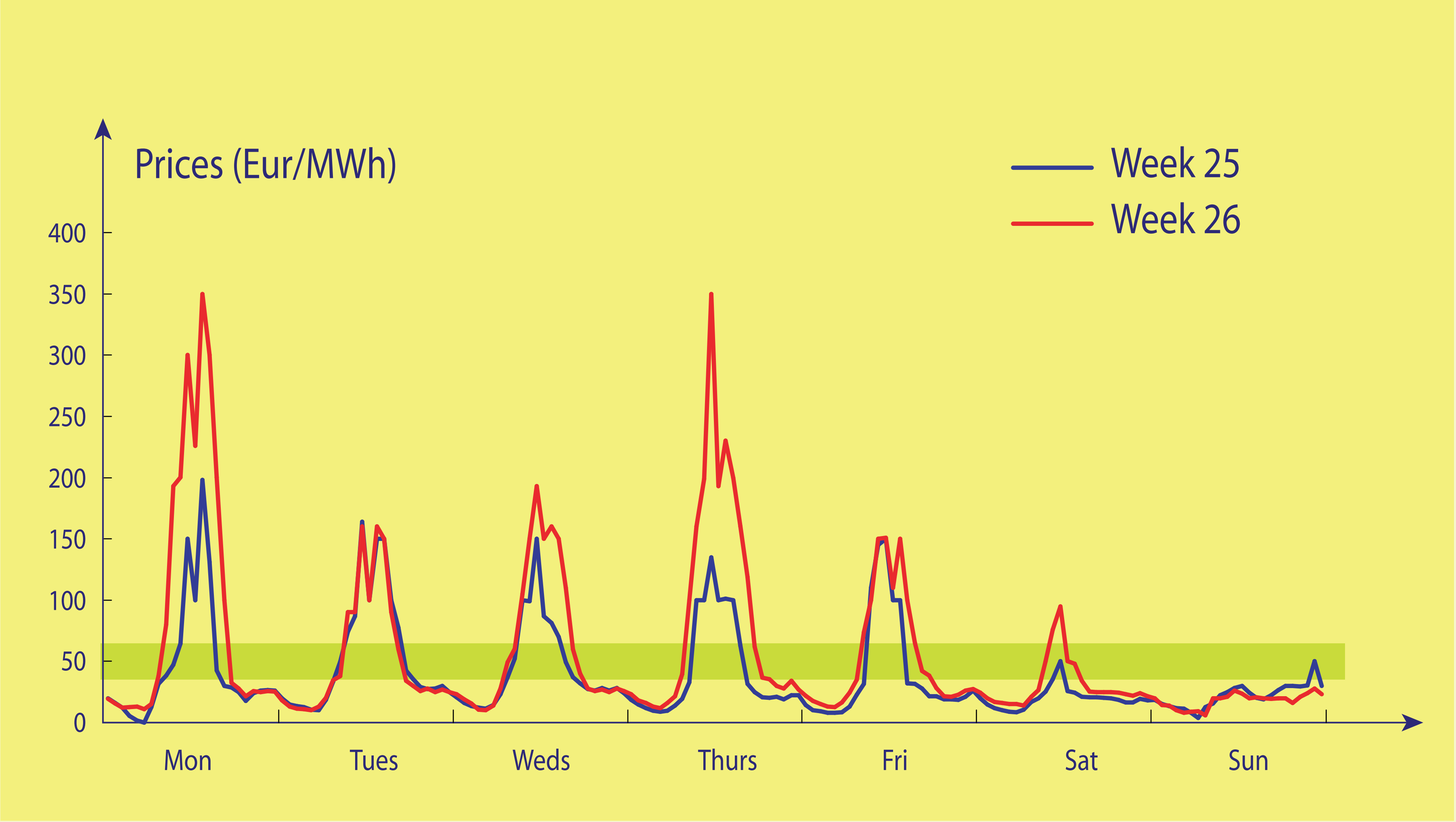
Lessons From the California "Apocalypse:" Jurisdiction Over Electric Utilities

Nicholas W. Fels and Frank R. Lindh

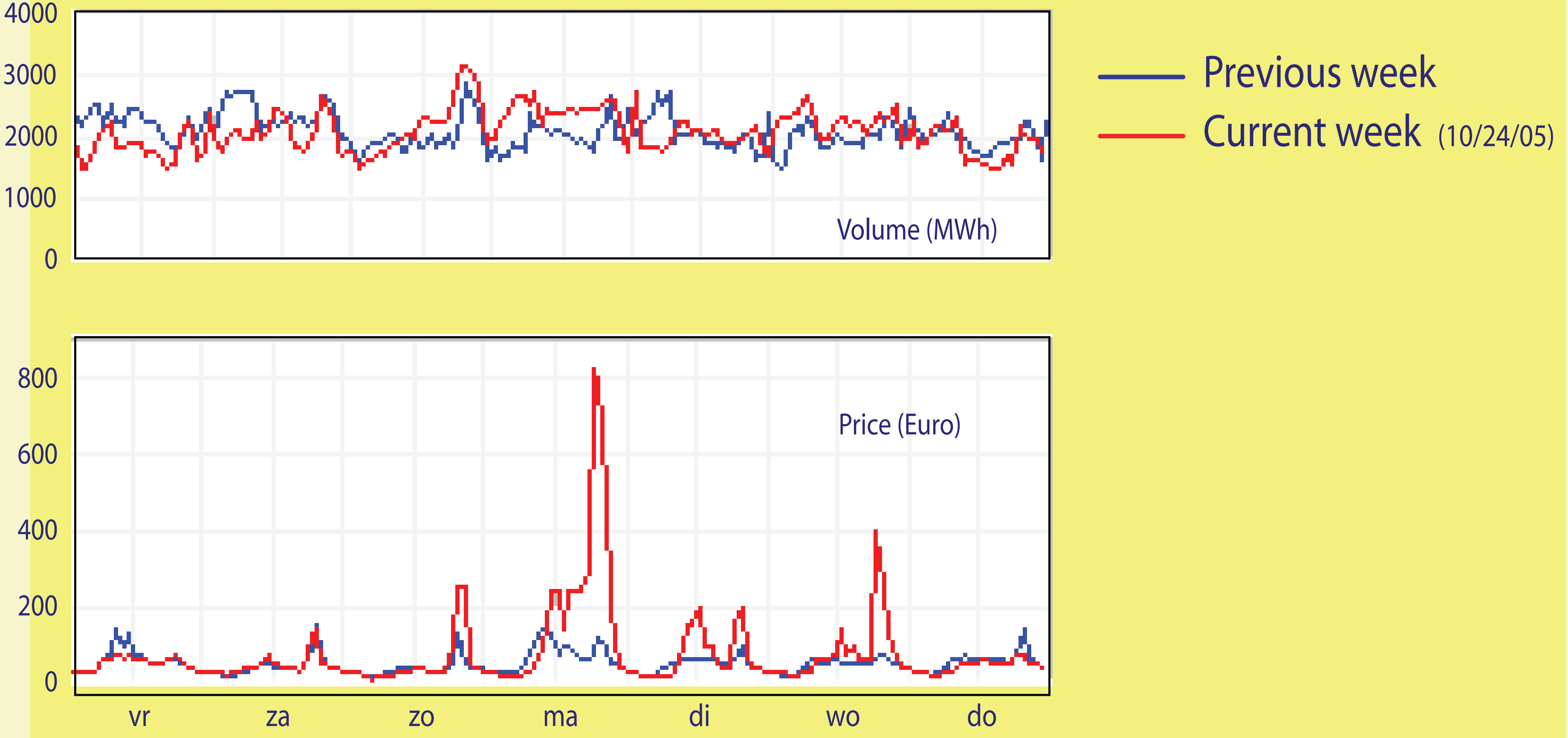
Energy Law Journal, Vol 22, No. 1, 2001

FERC ....authorized the ISO to "[reject]...bids in excess of whatever price levels it believes are appropriate ... file additional market-monitoring reports".

APX Europe, June 2003

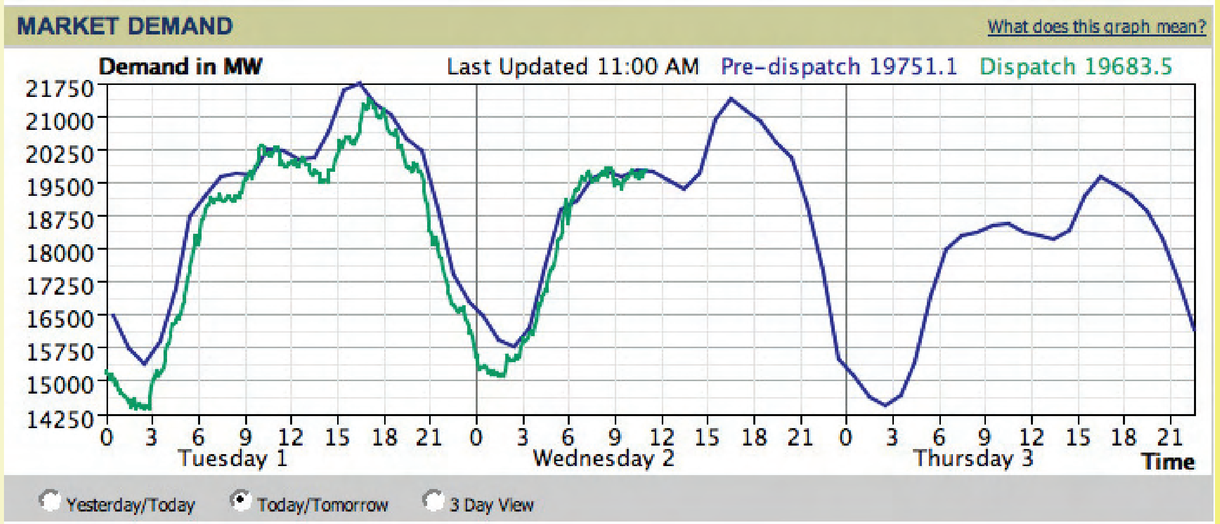


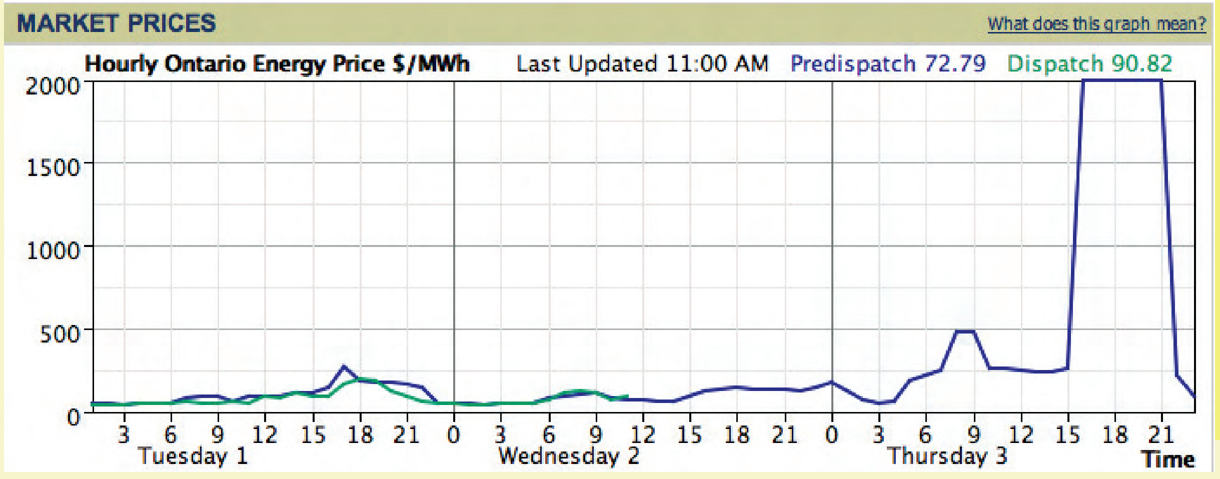
APX Europe, October 2005



Ontario, November 2005

ieso
Power to Ontario. On Demand.



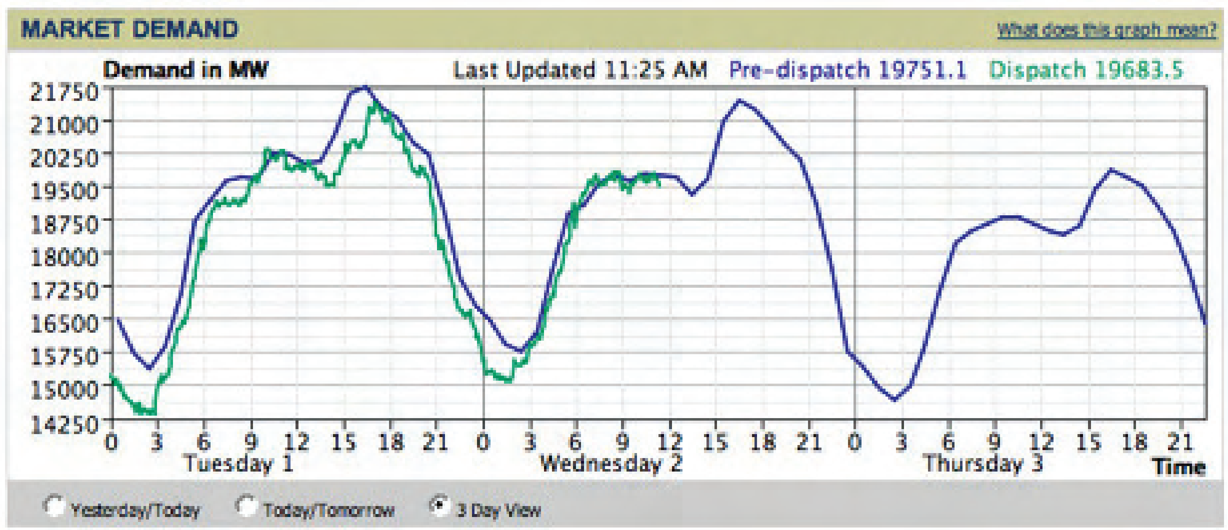


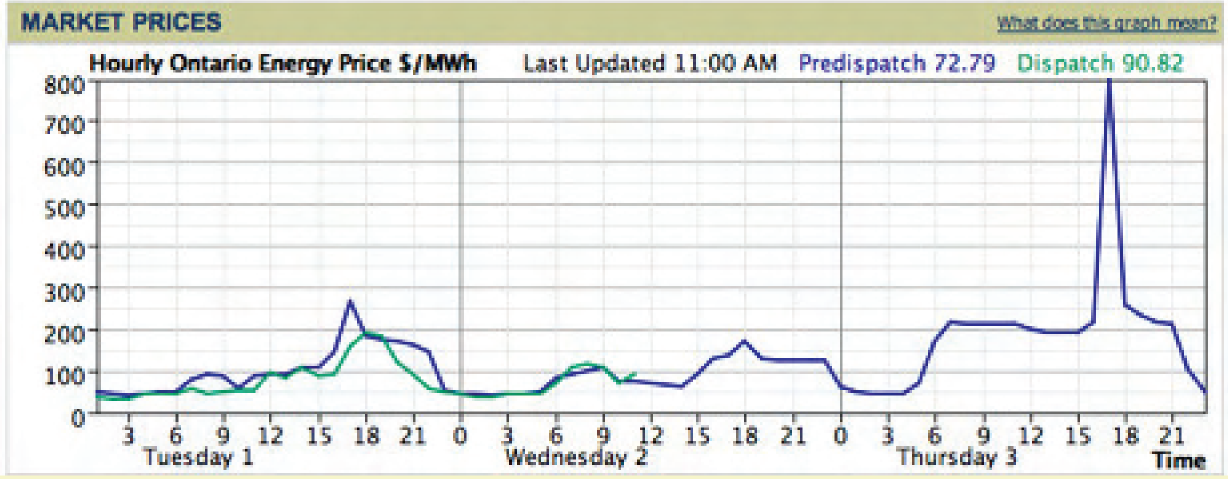
Ancillary service contract clause:
Minimum overall ramp rate of 50 MW/min.

Projected power prices reached $2000/MWh

Ontario, November 2005

ieso
Power to Ontario. On Demand.





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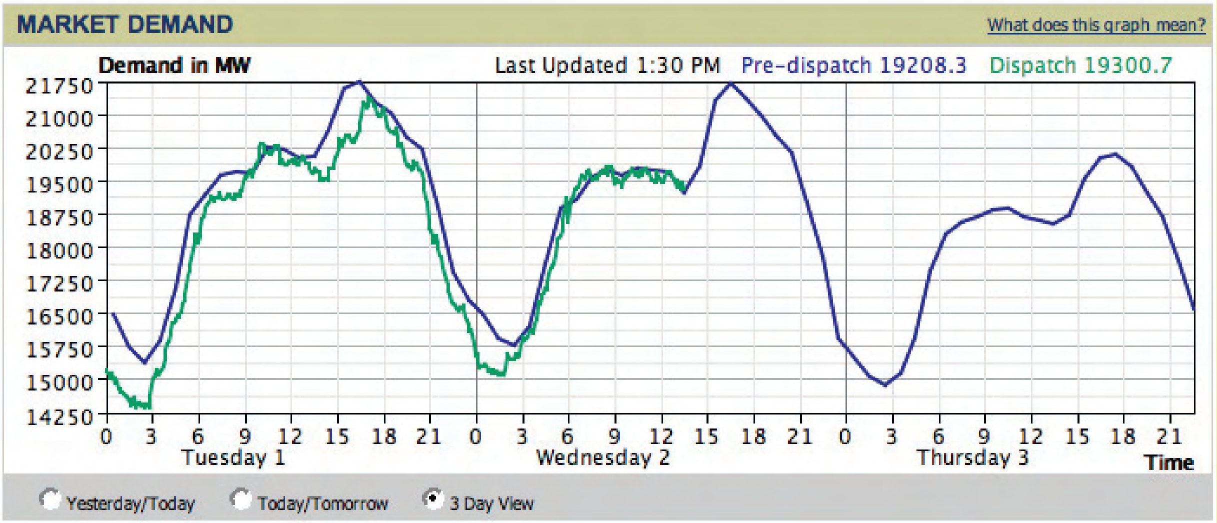

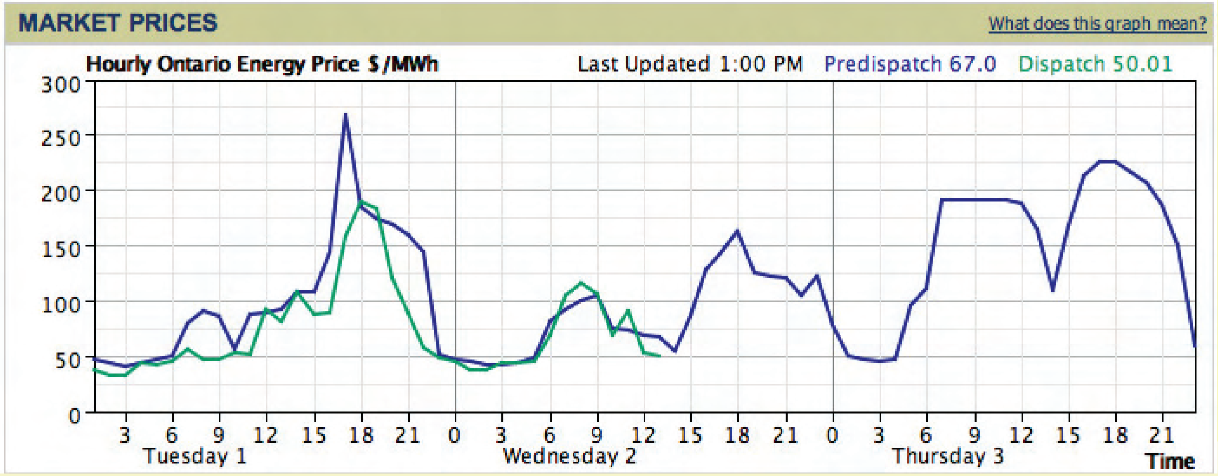

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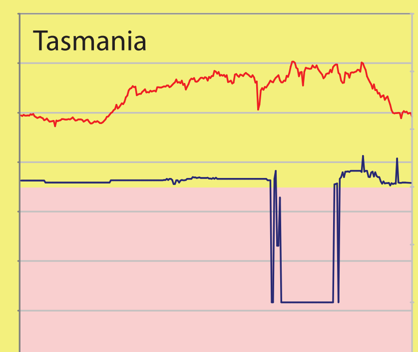
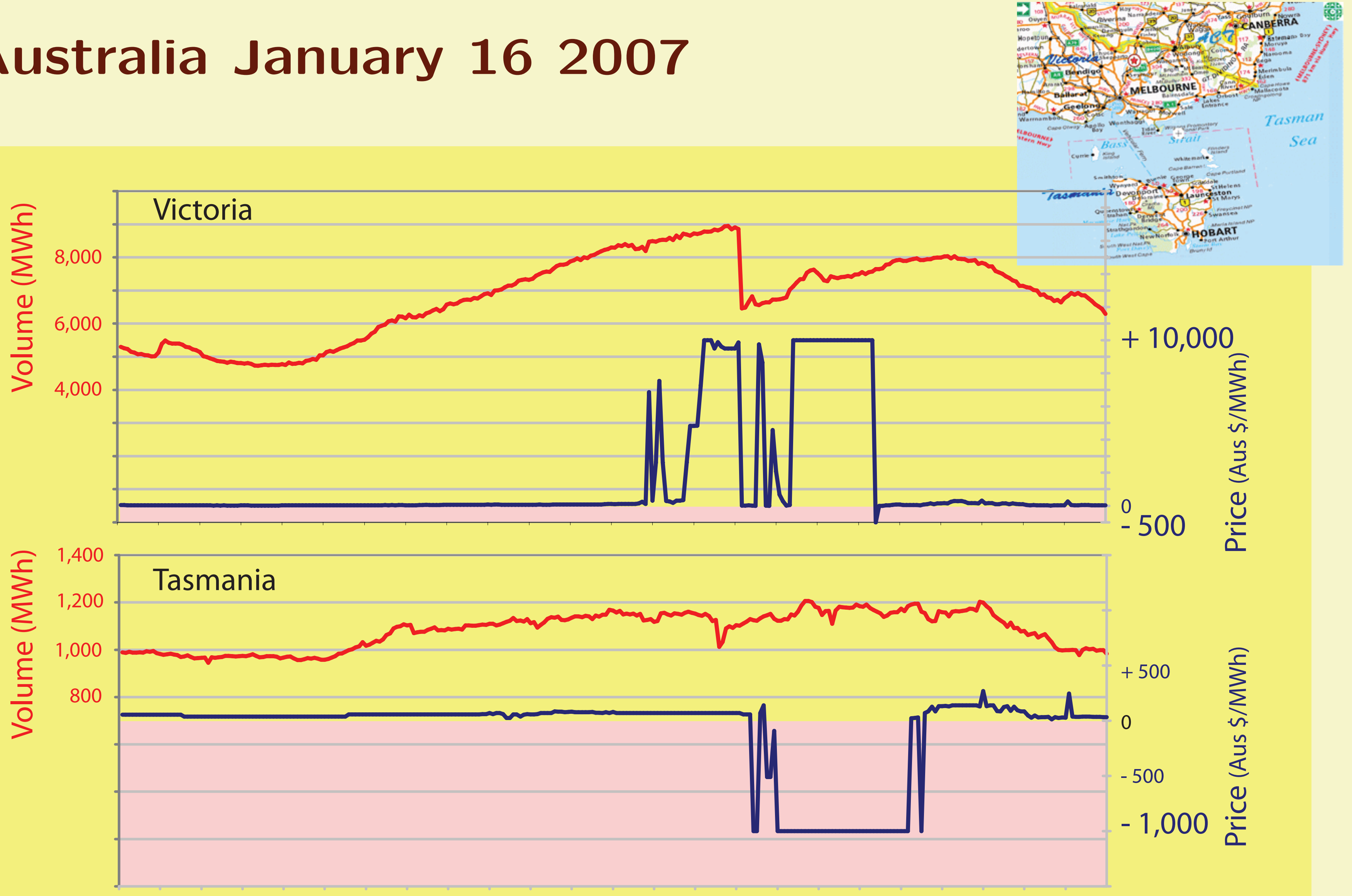
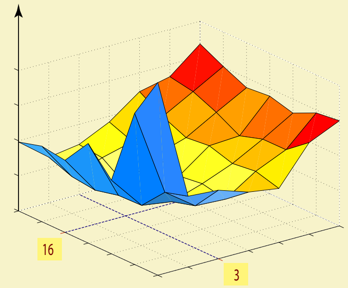
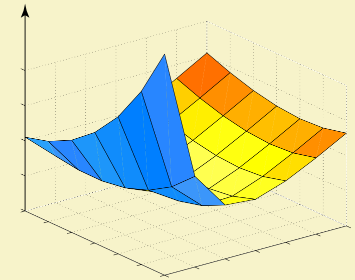
Australia January 16 2007



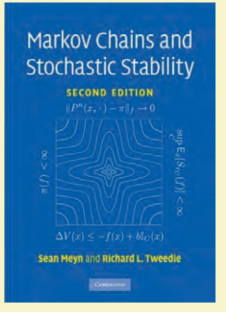
Australia January 16 2007

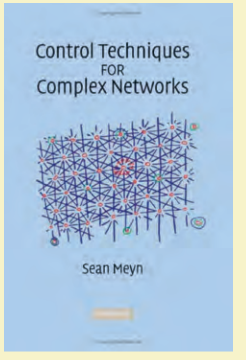






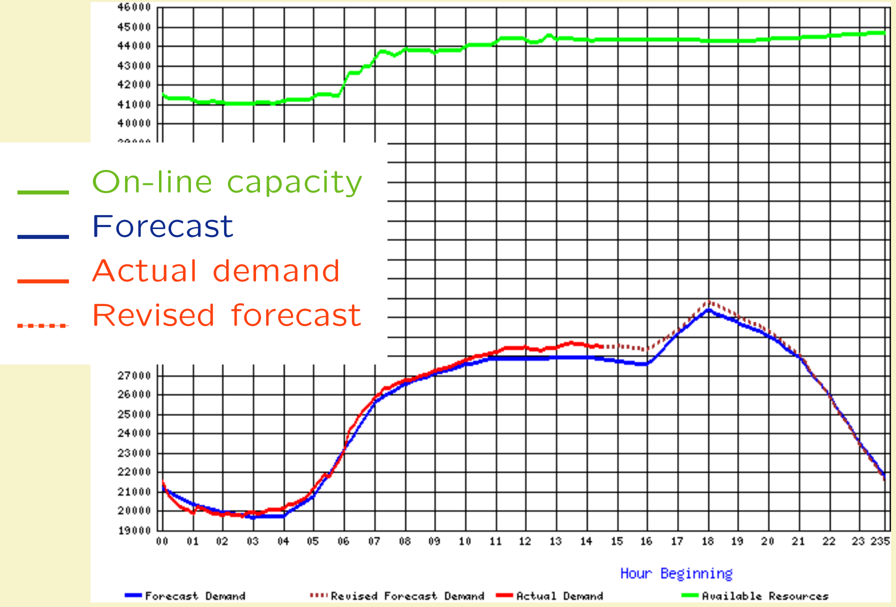
I
Centralized Control





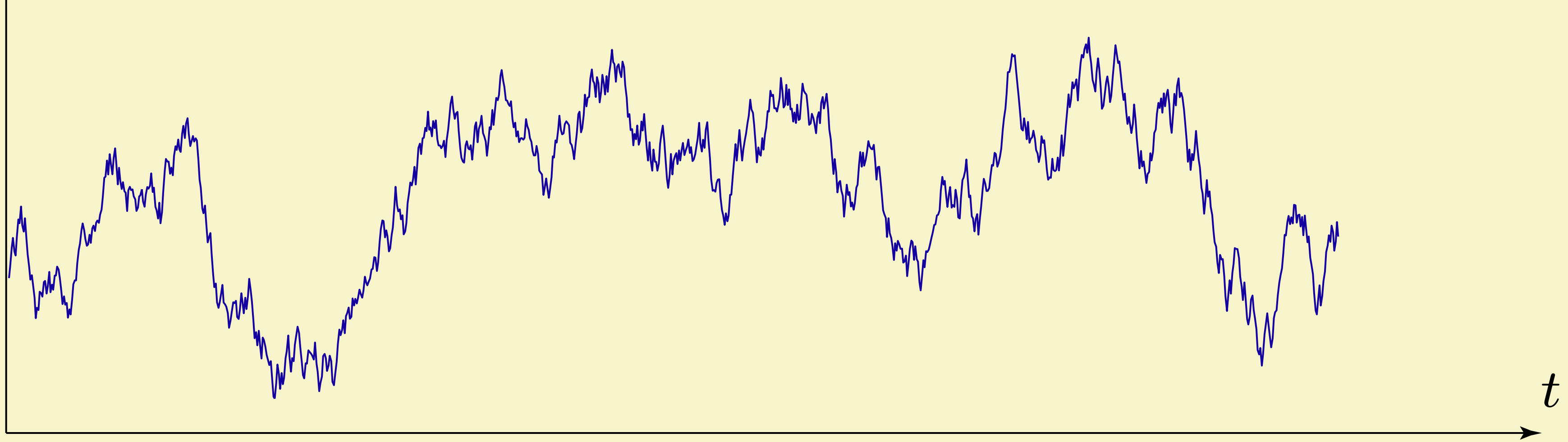
Dynamic model

Reserve options for services based on forecast statistics



Centered demand:

$D(t) =$ demand - forecast



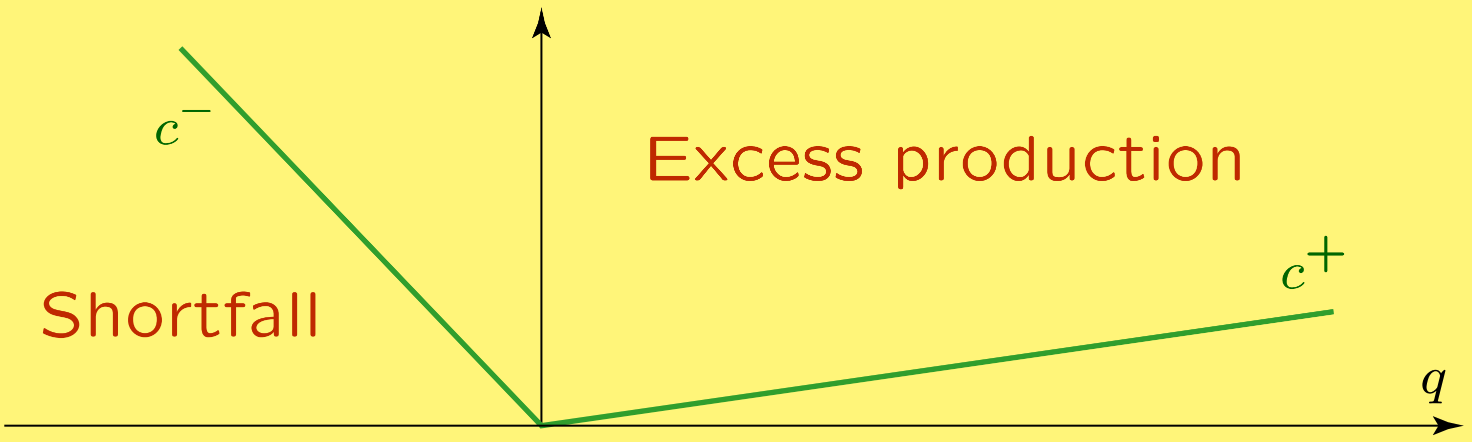
Dynamic Single-Commodity Model

Stochastic model: G Goods available at time t

D Normalized demand

Excess/shortfall: $Q(t) = G(t) - D(t)$

Normalized cost as a function of Q:



Scarf, Arrow, Bellman, ...

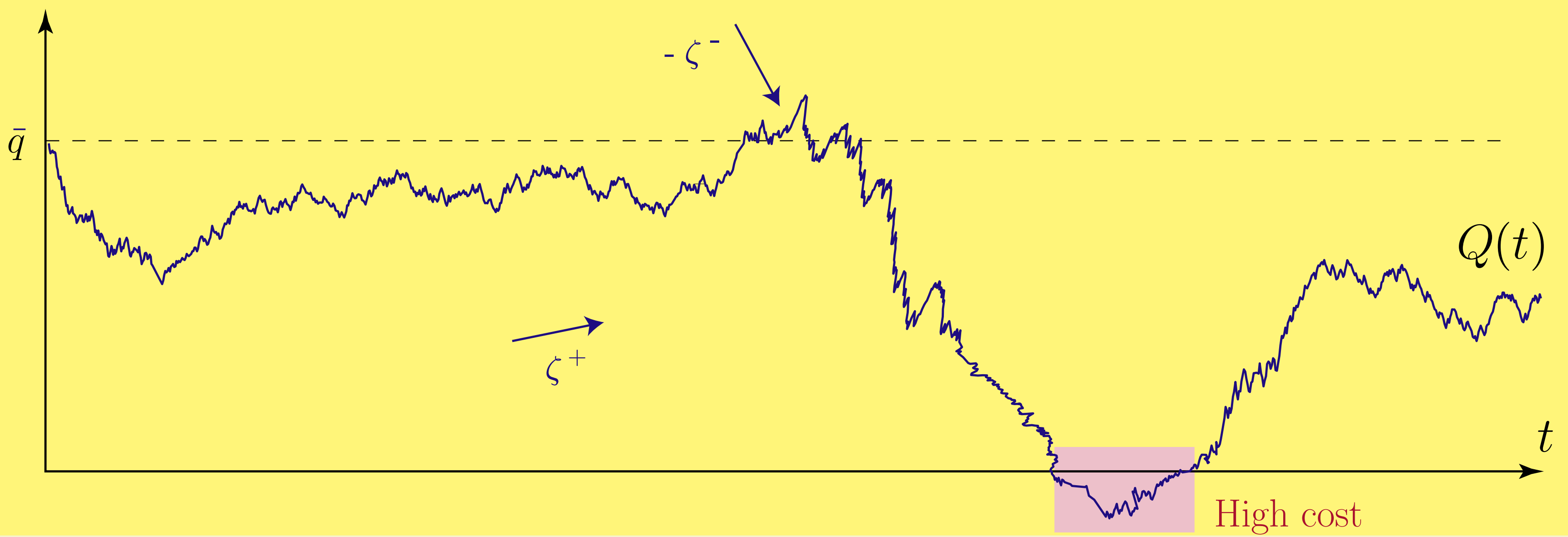
Dynamic Single-Commodity Model

Stochastic model: G Goods available at time t

D Normalized demand

Excess/shortfall: $Q(t) = G(t) - D(t)$

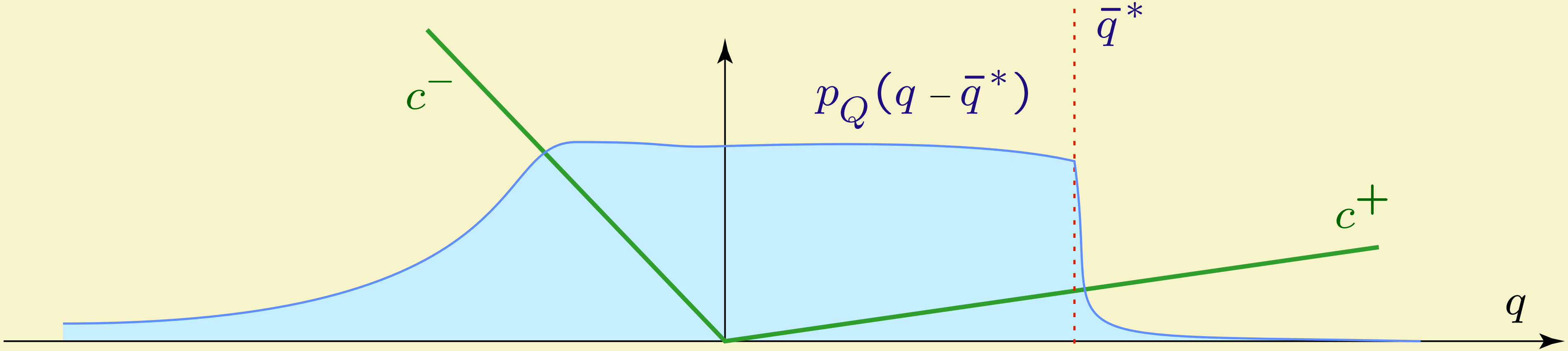
Generation is rate-constrained:



Dynamic Single-Commodity Model

Average cost: p_Q density when $\bar{q}^* = 0$

$$\mathsf{E}[c(Q)] = \int c(q - \bar{q}\,)\, p_Q(dq)$$



Optimal hedging-point: $\bar{q}^*$ solves

$$-c^- \mathsf{P}\{\, Q \le 0\} + c^+ \mathsf{P}\{\, Q \ge 0\} = 0$$

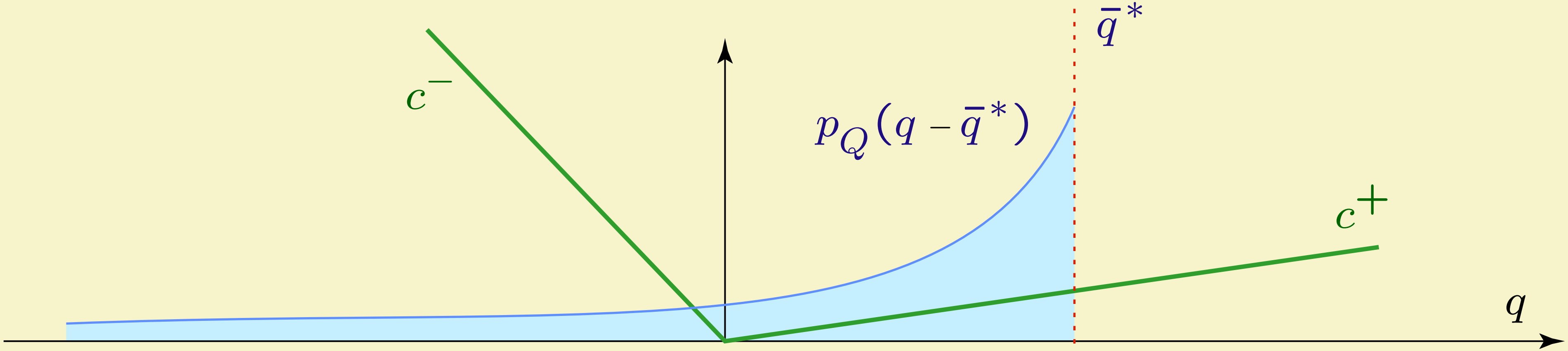
Dynamic Single-Commodity Model

Average cost: p_Q density when $\bar{q}^* = 0$

$$\mathsf{E}[c(Q)] = \int c(q - \bar{q})\, p_Q(dq)$$

RBM model:

p_Q exponential



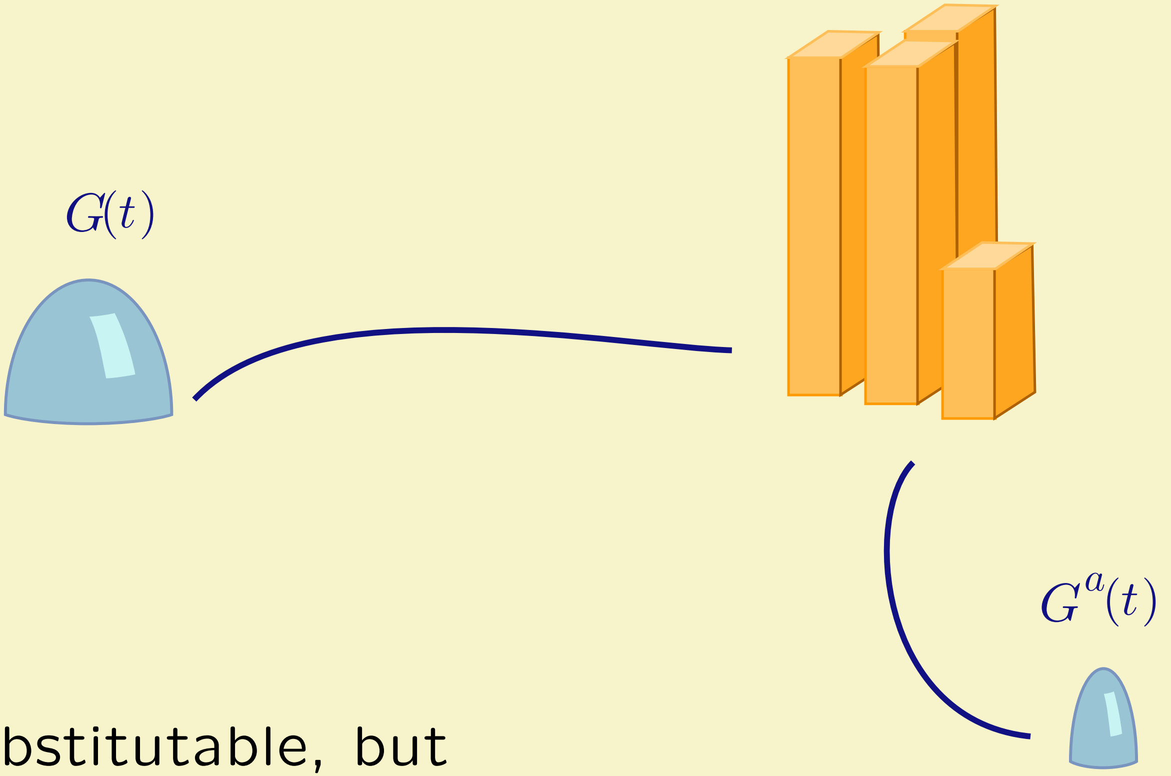
Optimal hedging-point: $\bar{q}^*$ solves

$$-c^- \mathsf{P}\{Q \le 0\} + c^+ \mathsf{P}\{Q \ge 0\} = 0$$

$$\bar{q}^* = \frac{1}{2} \frac{\sigma^2}{\zeta^+} \log \frac{c^-}{c^+}$$

Wein '92

Ancillary service

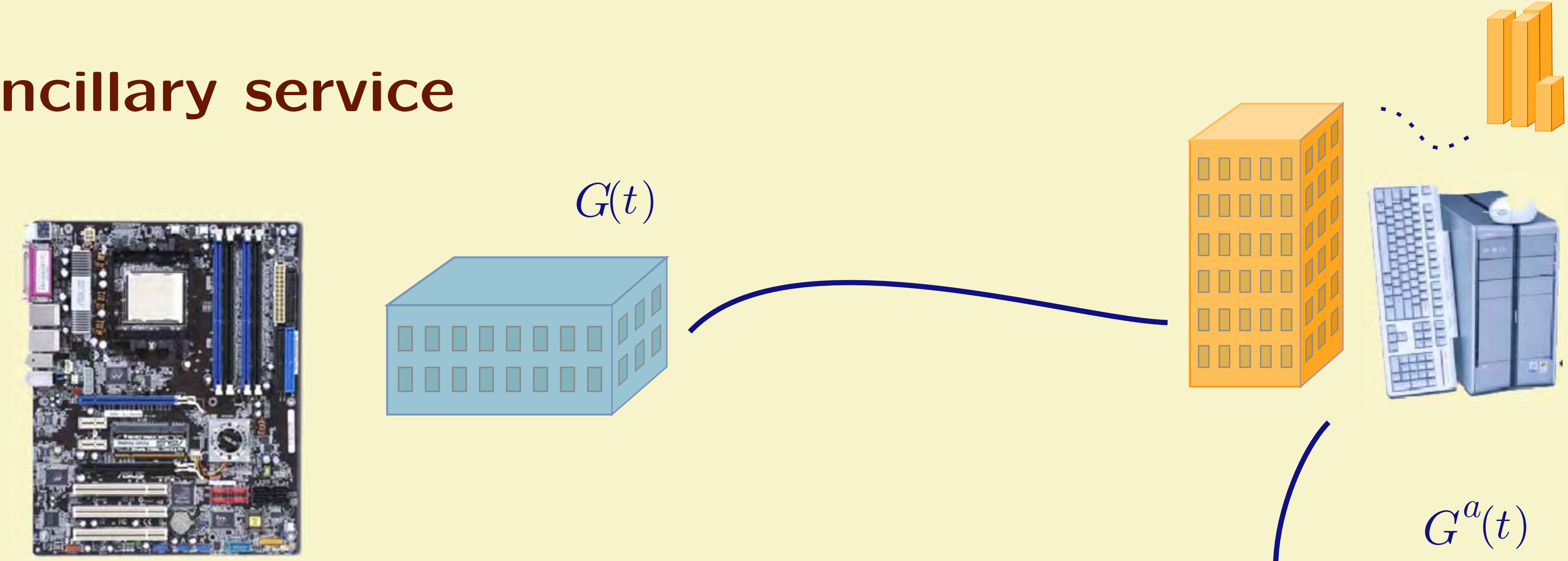


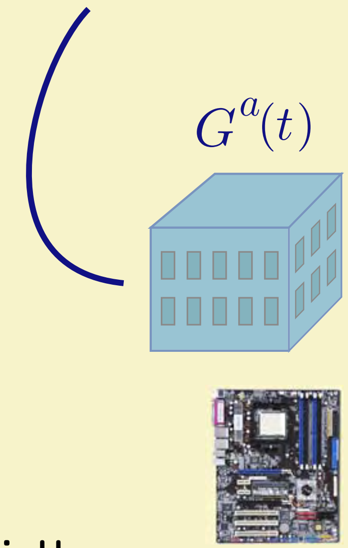
The two goods are substitutable, but

1. primary service is available at a lower price

2. ancillary service can be ramped up more rapidly

Ancillary service





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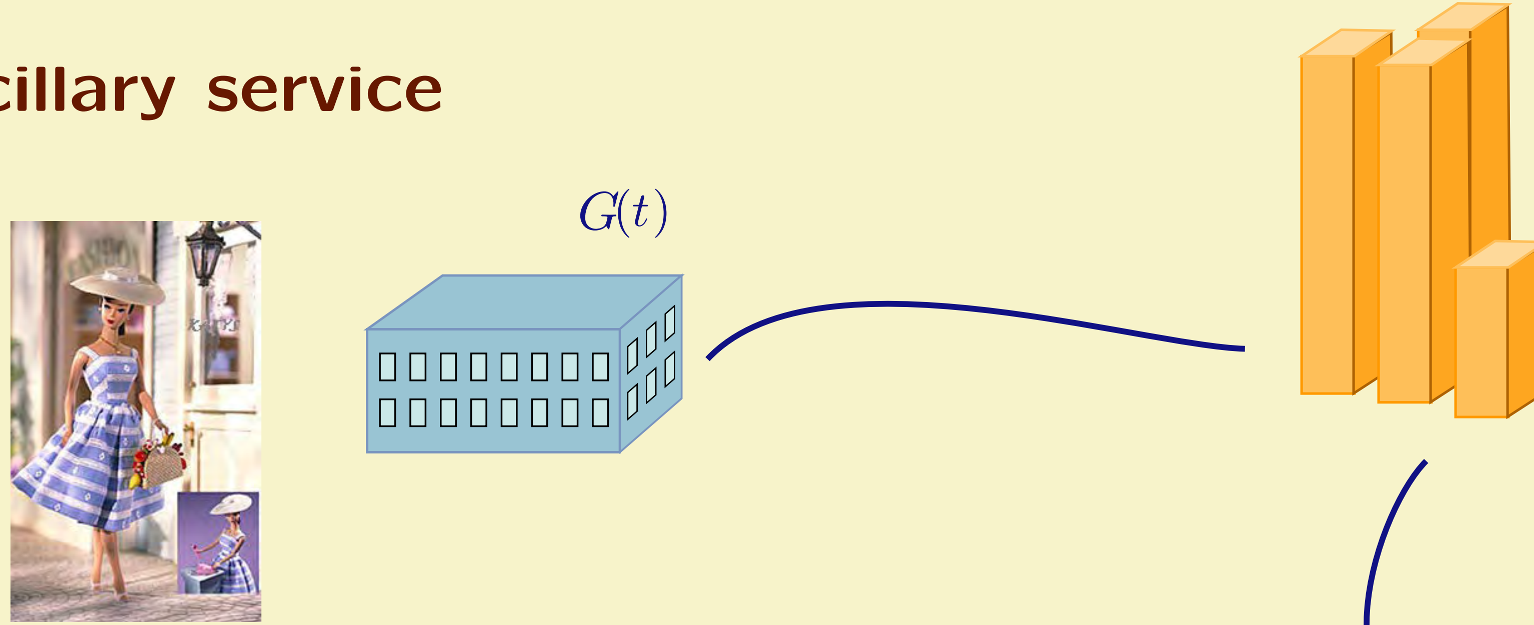

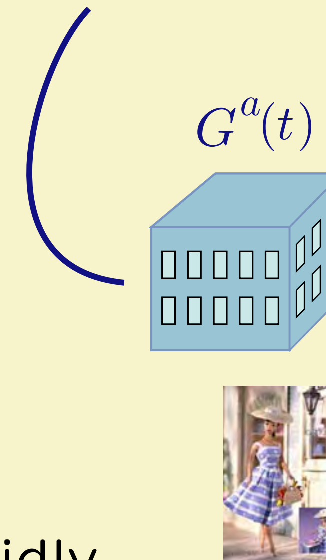

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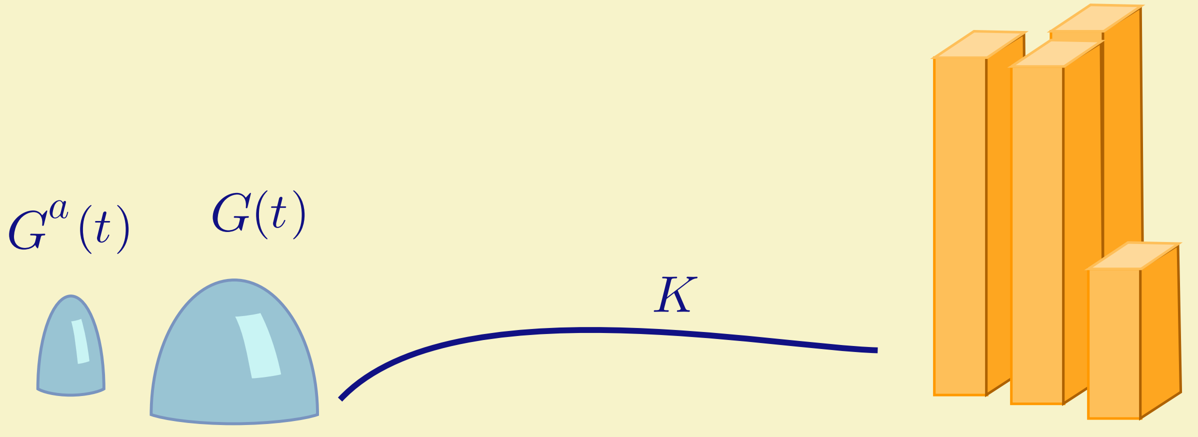
Ancillary service



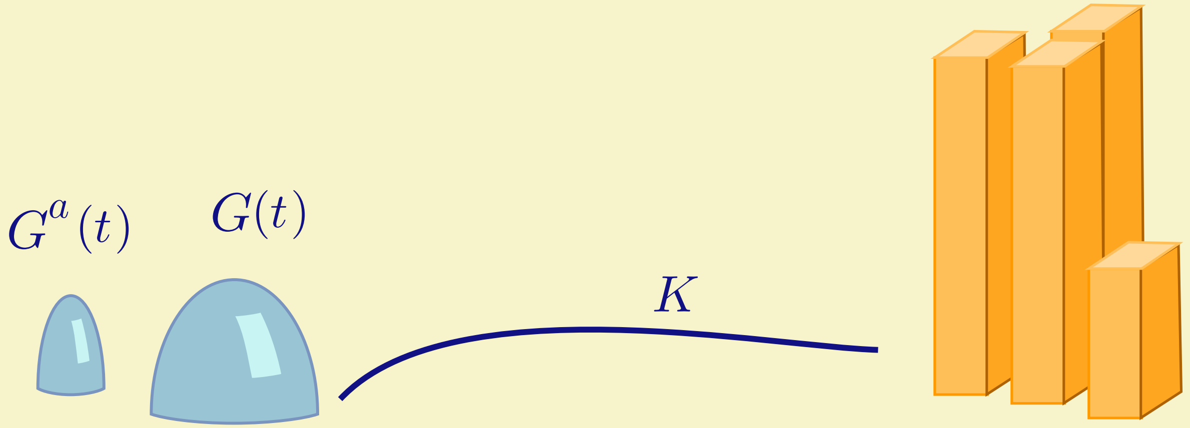
Excess capacity:

$$Q(t) = G(t) + G^a(t) - D(t), \quad t \geq 0\,.$$

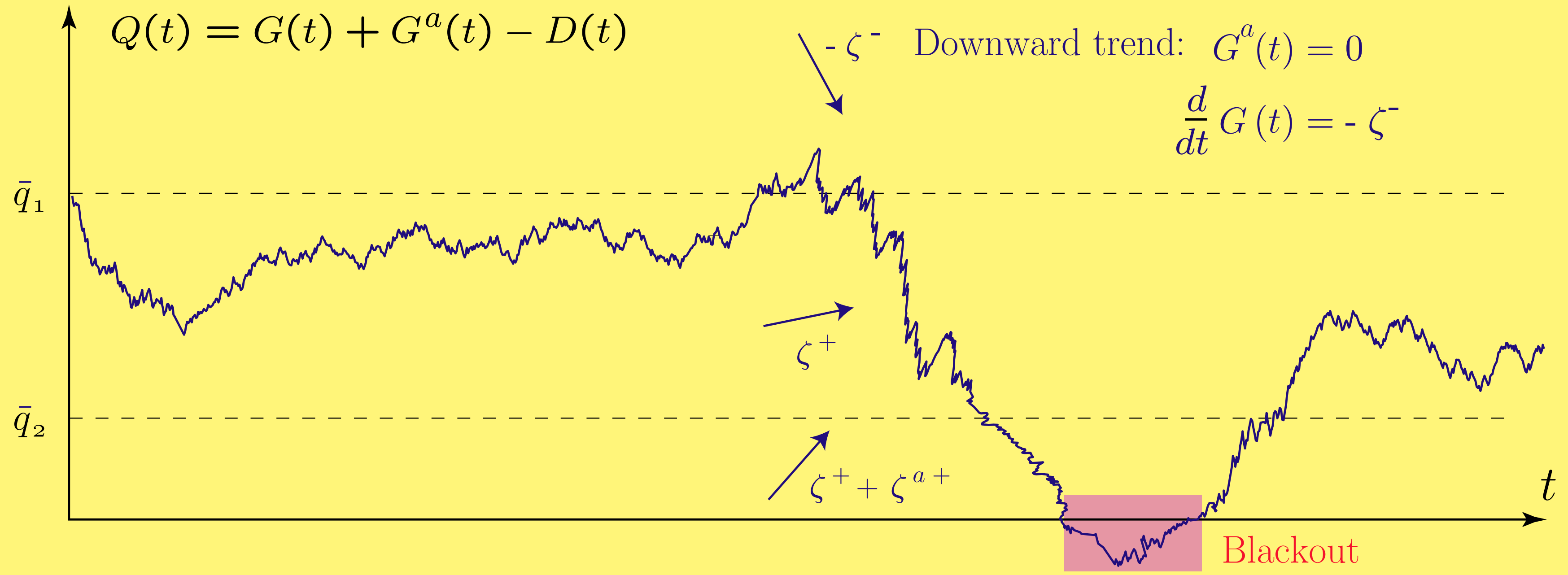
Power flow subject to peak and rate constraints:

$$-\zeta^{a-} \leq \frac{d}{dt}G^a(t) \leq \zeta^{a+} \qquad\qquad -\zeta^{-} \leq \frac{d}{dt}G(t) \leq \zeta^{+}$$

Ancillary service



Policy: *hedging policy with multiple thresholds*



Diffusion model & control $\quad X(t) = \binom{Q(t)}{G^a(t)}$

Relaxations: instantaneous ramp-down rates:

$$-\infty \le \frac{d}{dt} G(t) \le \zeta^+, \quad -\infty \le \frac{d}{dt} G^a(t) \le \zeta^{a+}.$$

Cost structure:

$$c(X(t)) = c_1 G(t) + c_2 G^a(t) + c_3 |Q(t)| \mathbf{1}\{Q(t) < 0\}$$

Control: design hedging points to minimize average-cost,

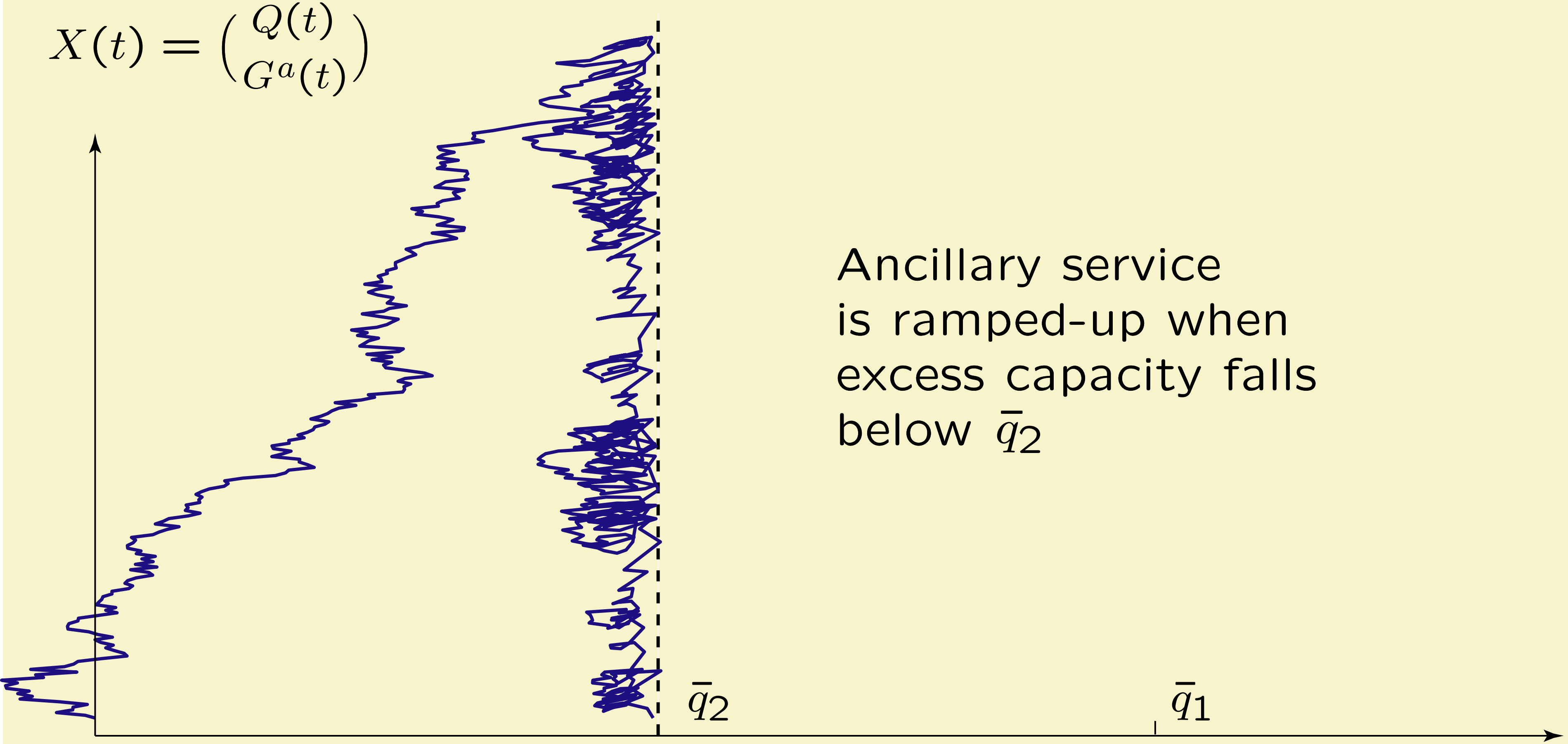
$$\min \mathsf{E}_\pi \left[c(Q(t)) \right].$$

Diffusion model & control

Markov model:

Hedging-point policy:

$$X(t) = \binom{Q(t)}{G^a(t)}$$

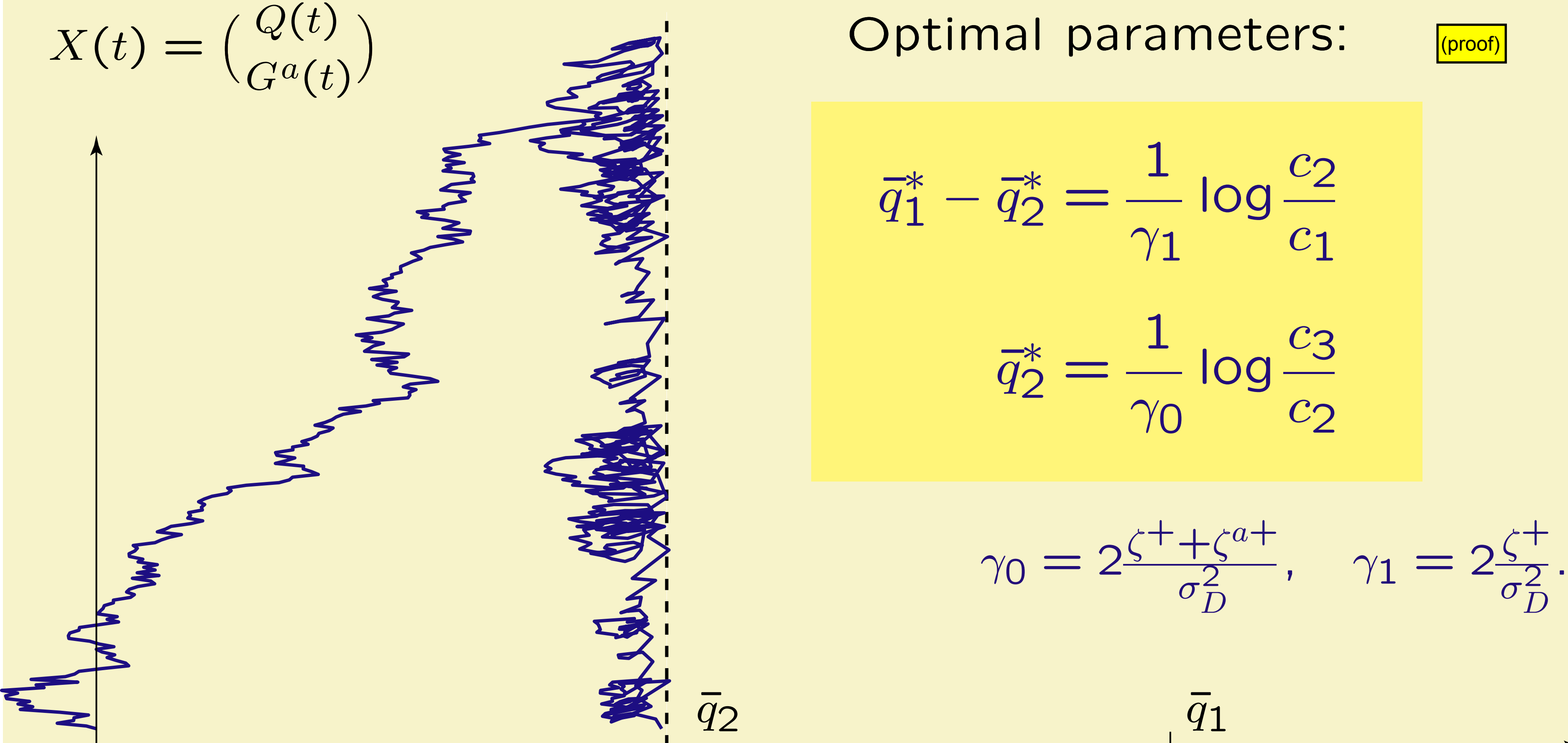


Ancillary service is ramped-up when excess capacity falls below $\bar{q}_2$

Markov model & control

Markov model:

$$X(t) = \begin{pmatrix} Q(t) \\ G^a(t) \end{pmatrix}$$



Hedging-point policy:

Optimal parameters: (proof)

$$\bar{q}_1^* - \bar{q}_2^* = \frac{1}{\gamma_1} \log \frac{c_2}{c_1}$$

$$\bar{q}_2^* = \frac{1}{\gamma_0} \log \frac{c_3}{c_2}$$

$$\gamma_0 = 2\frac{\zeta^+ + \zeta^{a+}}{\sigma_D^2}, \quad \gamma_1 = 2\frac{\zeta^+}{\sigma_D^2}.$$

Simulation

Discrete Markov model:

$$Q(k+1) - Q(k) = \zeta(k) + \zeta^a(k) + \mathcal{E}(k+1)$$

$\mathcal{E}(k)$ i.i.d. Bernoulli.

$\zeta(k)$, $\zeta^a(k)$ allocation increments.

Optimal hedging-points for RBM:

$$\bar{q}_1 - \bar{q}_2 = 14.978$$

$$\bar{q}_2 = 2.996$$

Simulation

Discrete Markov model:

$$Q(k+1) - Q(k) = \zeta(k) + \zeta^a(k) + \mathcal{E}(k+1)$$

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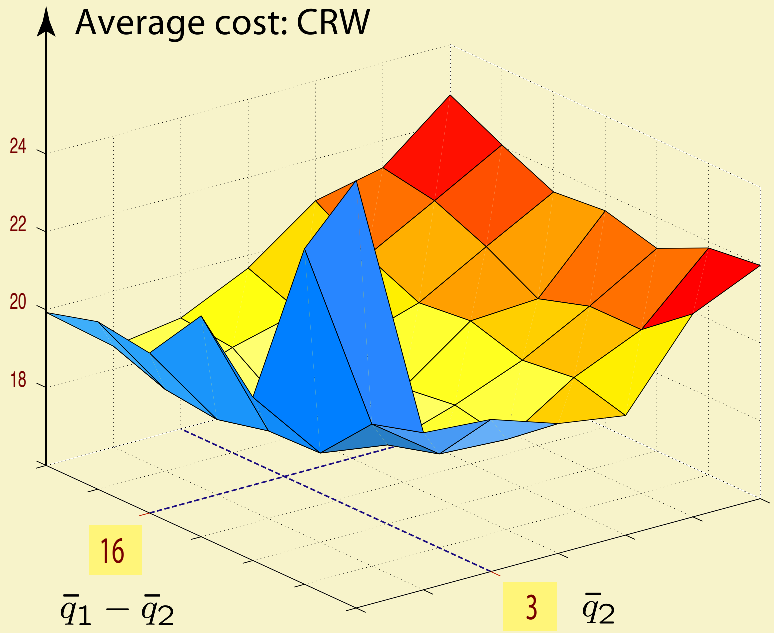
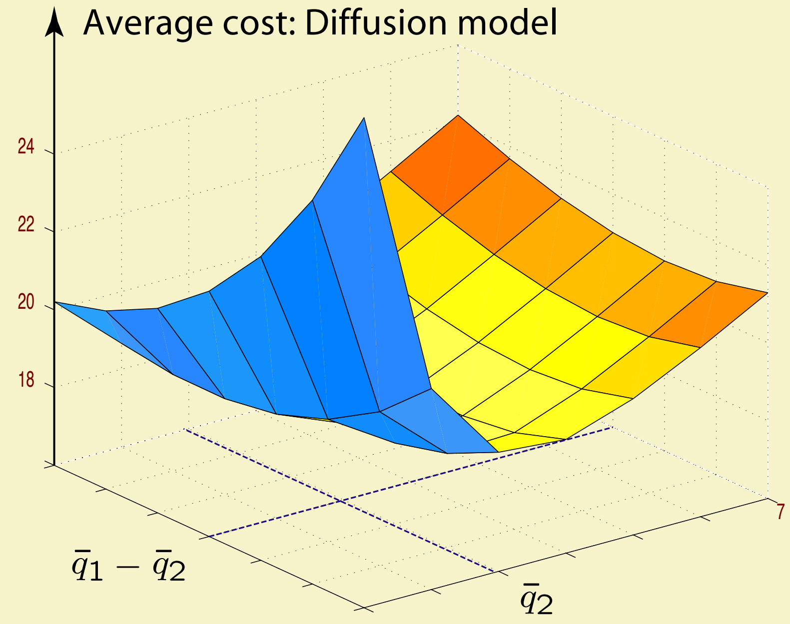
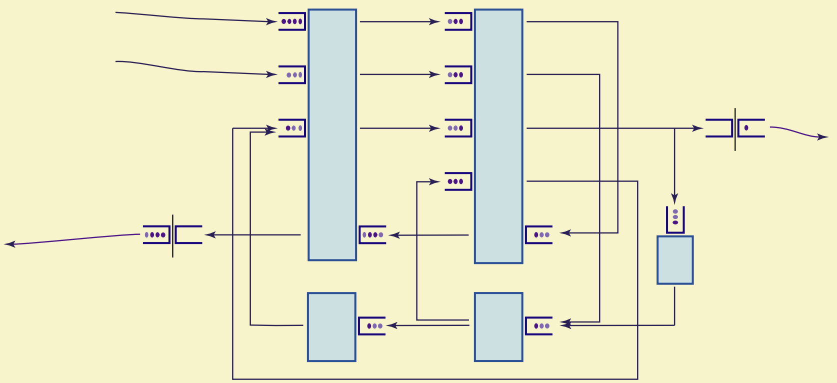


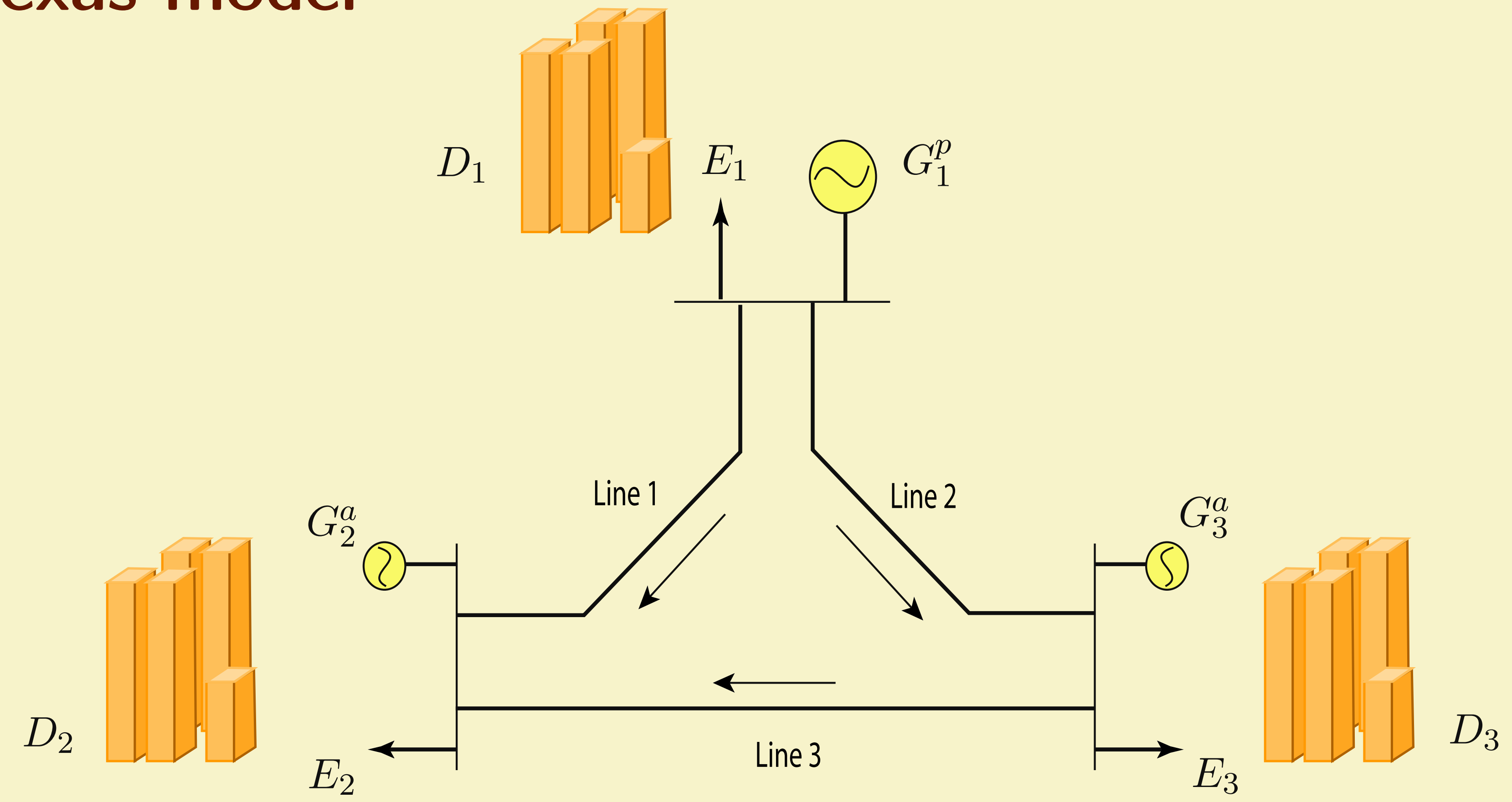




II
Relaxations

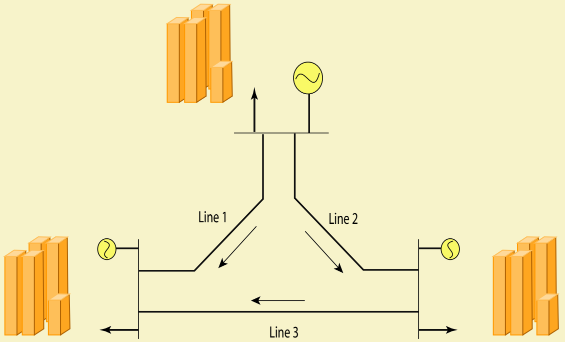
(skip to market)

Texas model



Resource pooling from San Antonio to Houston?

Aggregate model



$$Q_A(t) = \text{extraction - demand} \quad = \quad \sum E_i(t) - D_i(t)$$

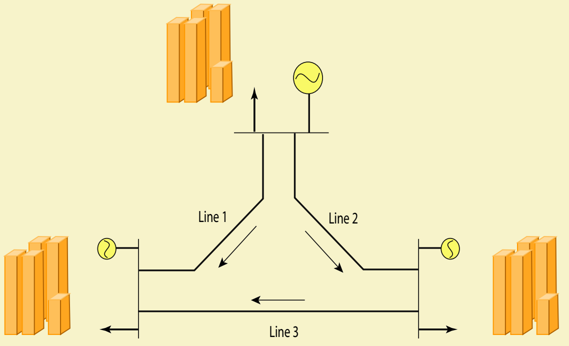
$$G_A^a(t) = \text{ aggregate ancillary} \quad = \quad \sum G_i^a(t)$$

Assume Brownian demand, rate constraints as before

Provided there are no transmission constraints,

$$X_A = (Q_A, G_A^a) \equiv \text{ single producer/consumer model}$$

Effective cost $\bar{c}(x_A, d)$



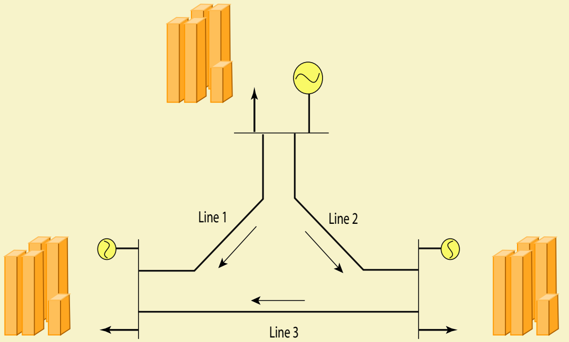
Given demand and aggregate state
find the cheapest consistent
network configuration subject to transmission constraints

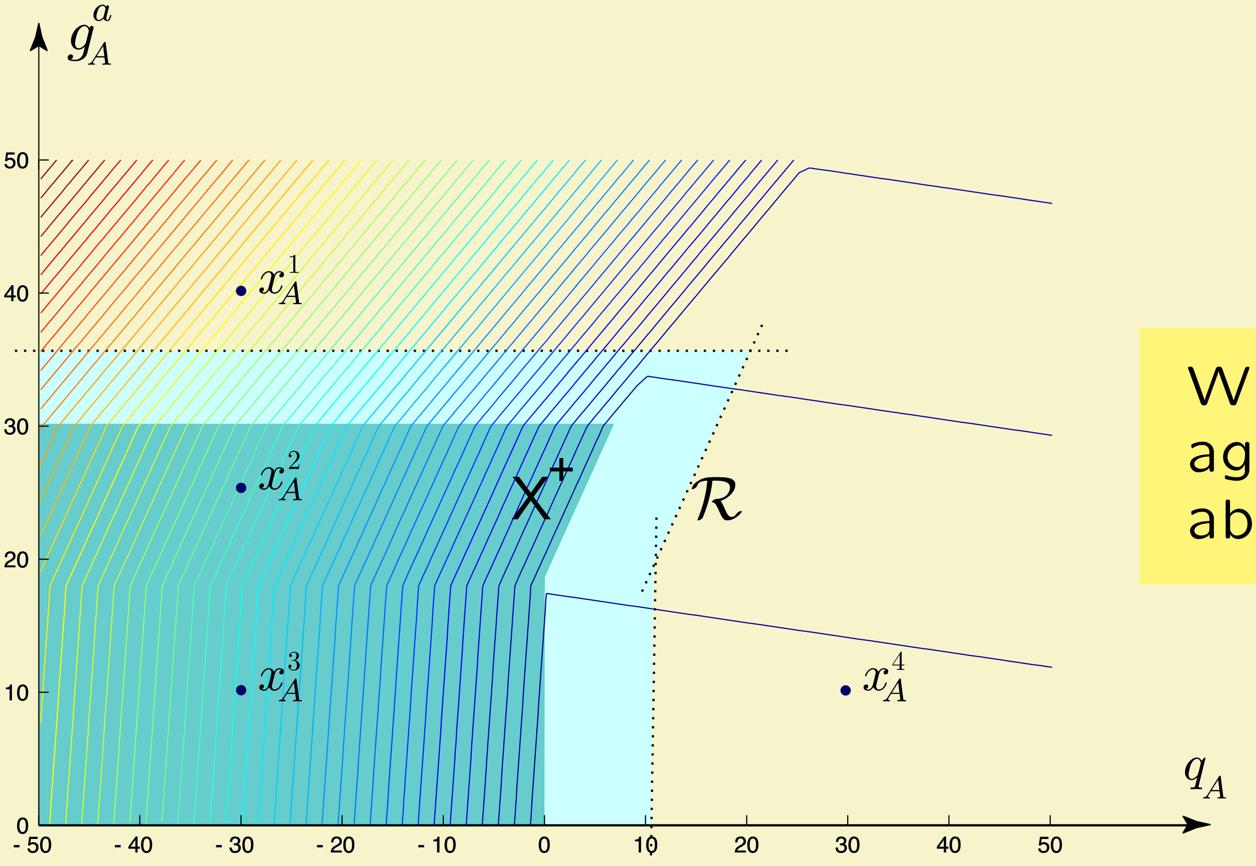
min $\sum (c_i^p g_i^p + c_i^a g_i^a + c_i^{bo} q_i^-)$

s.t.

$q_A = \sum(e_i - d_i)$ $g_A^a = \sum g_i^a$	*consistency*
$0 = \sum(g_i^p + g_i^a - e_i)$	*extraction = generation*
$q = e - d$	*vector reserves*
$f = \Delta p$	*power flow equations*
$f \in \mathsf{F}$	*transmission constraints*

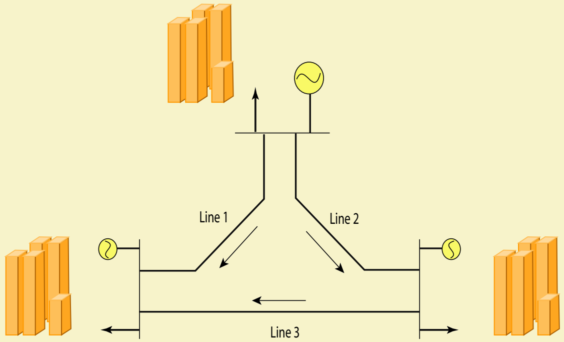
Effective cost $\bar{c}(x_A, d)$

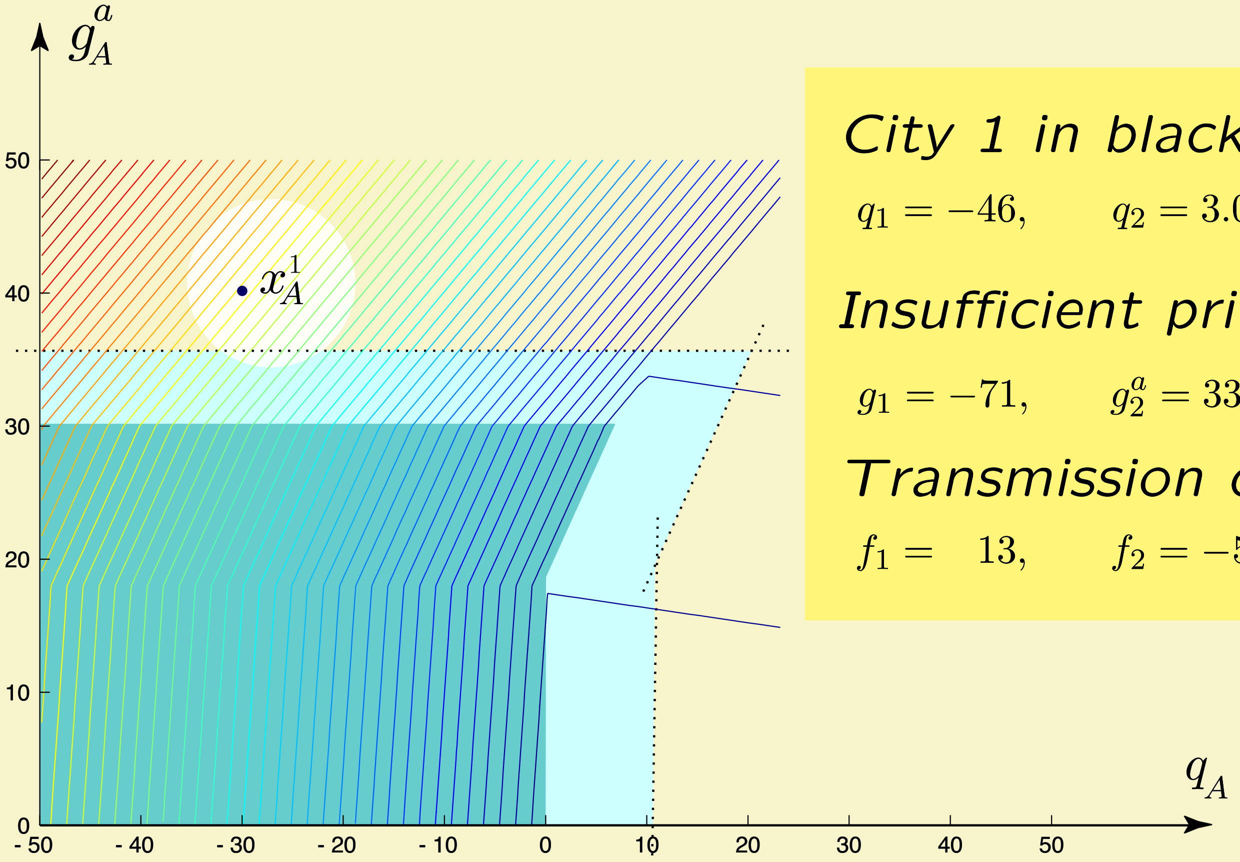




What do these aggregate states say about the network?

Effective cost $\bar{c}(x_A, d)$





City 1 in blackout:

$q_1 = -46, \qquad q_2 = 3.0564, \qquad q_3 = 12.9436$

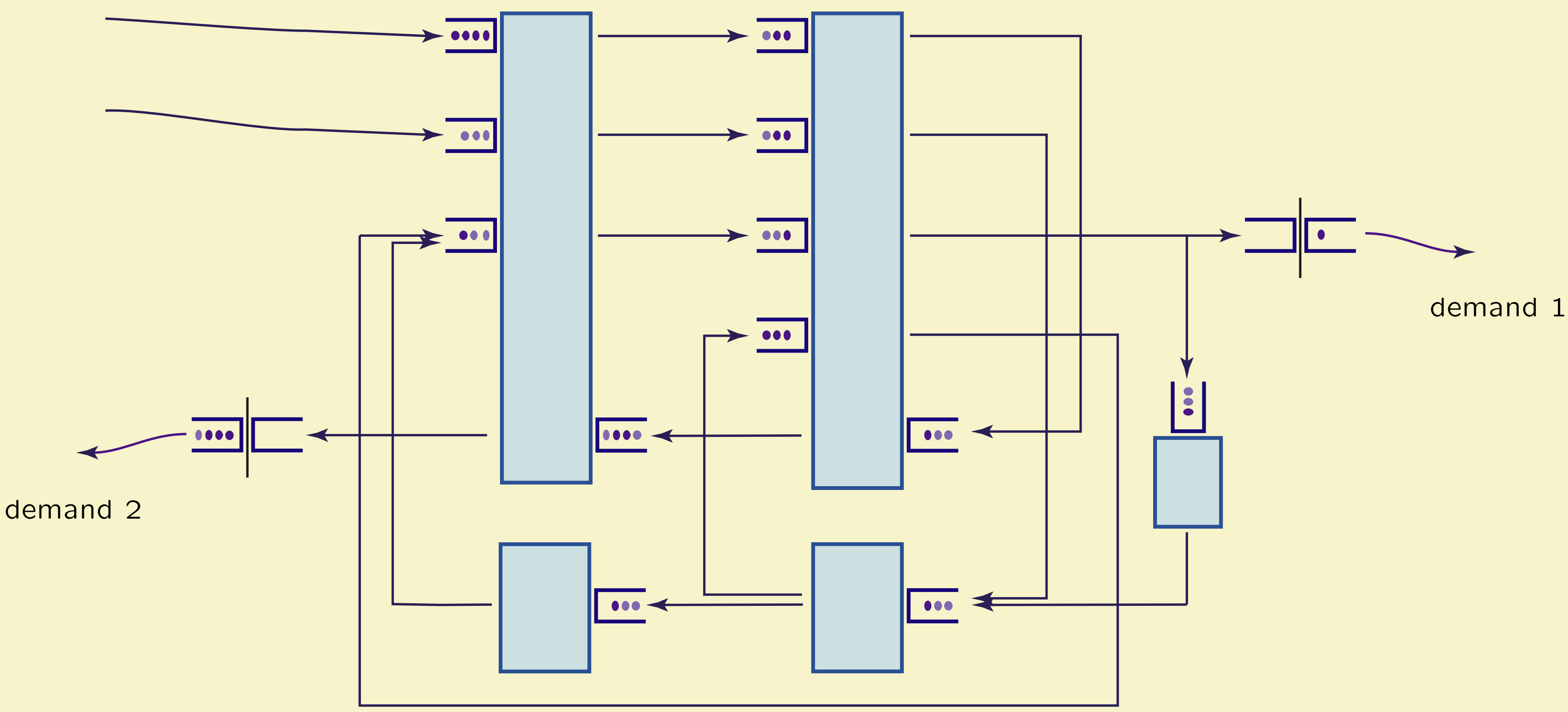
Insufficient primary generation:

$g_1 = -71, \qquad g_2^a = 33.0564, \qquad g_3^a = 6.9436$

Transmission constraints binding:

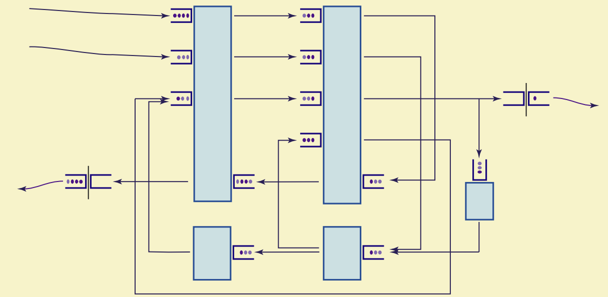
$f_1 = \; 13, \qquad f_2 = -5, \qquad f_3 = -8$

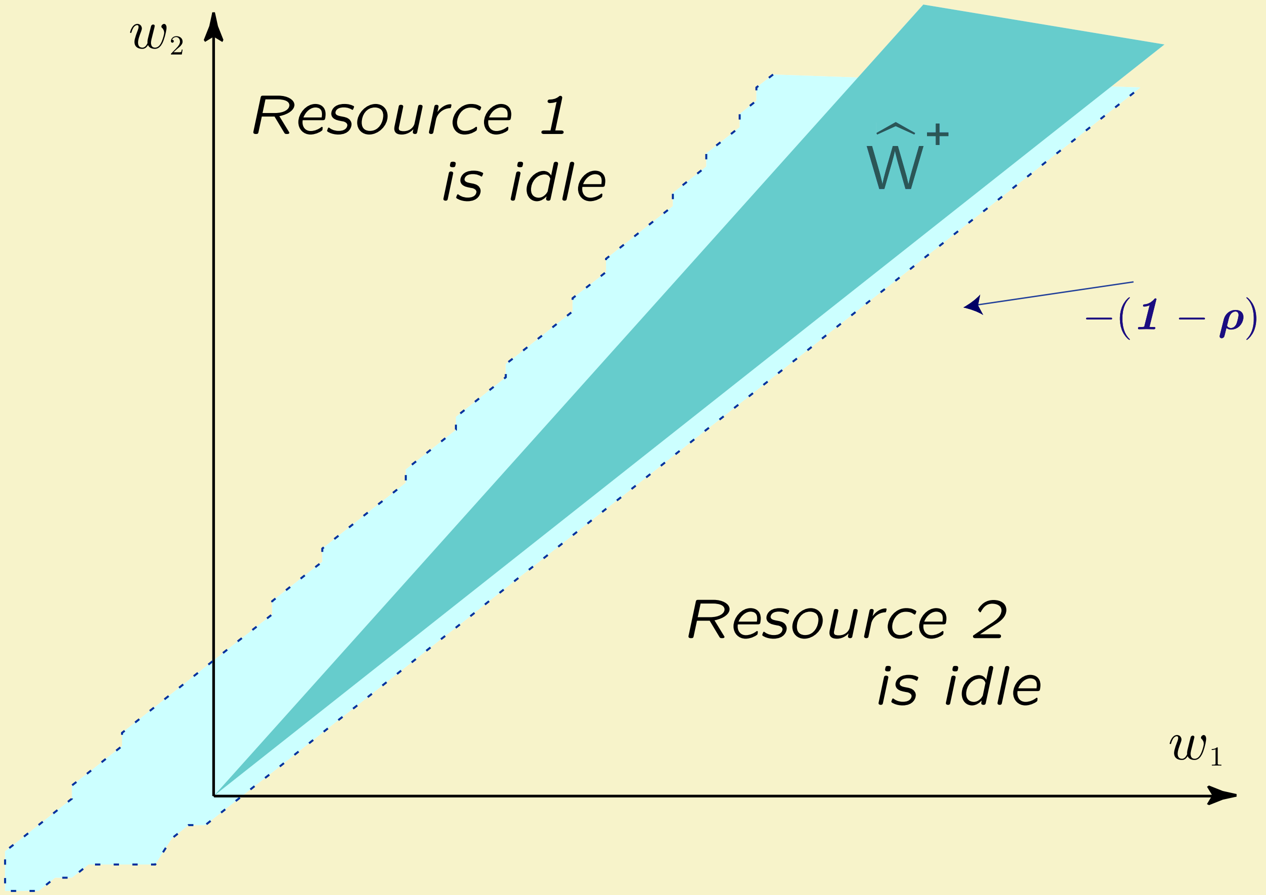
Inventory model



Controlled work-release, controlled routing,
uncertain demand.

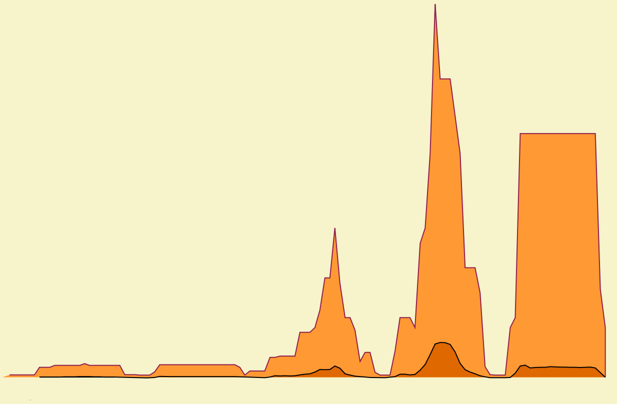
Inventory model: Workload relaxation





Asymptotes:

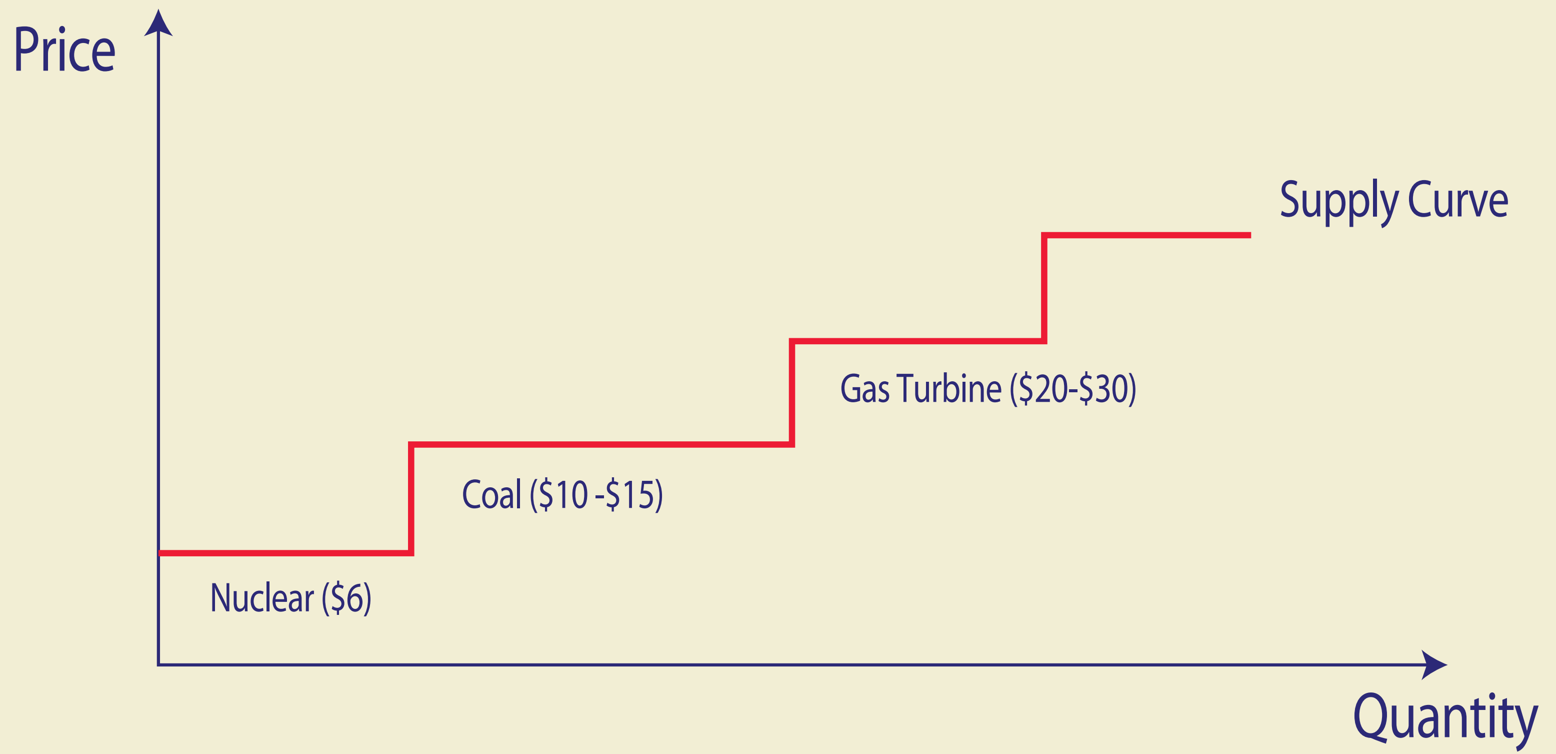
$$\bar{q}^* = \frac{1}{2}\frac{\sigma^2}{\zeta^+}\log\frac{c^-}{c^+}$$



III
Decentralized Control

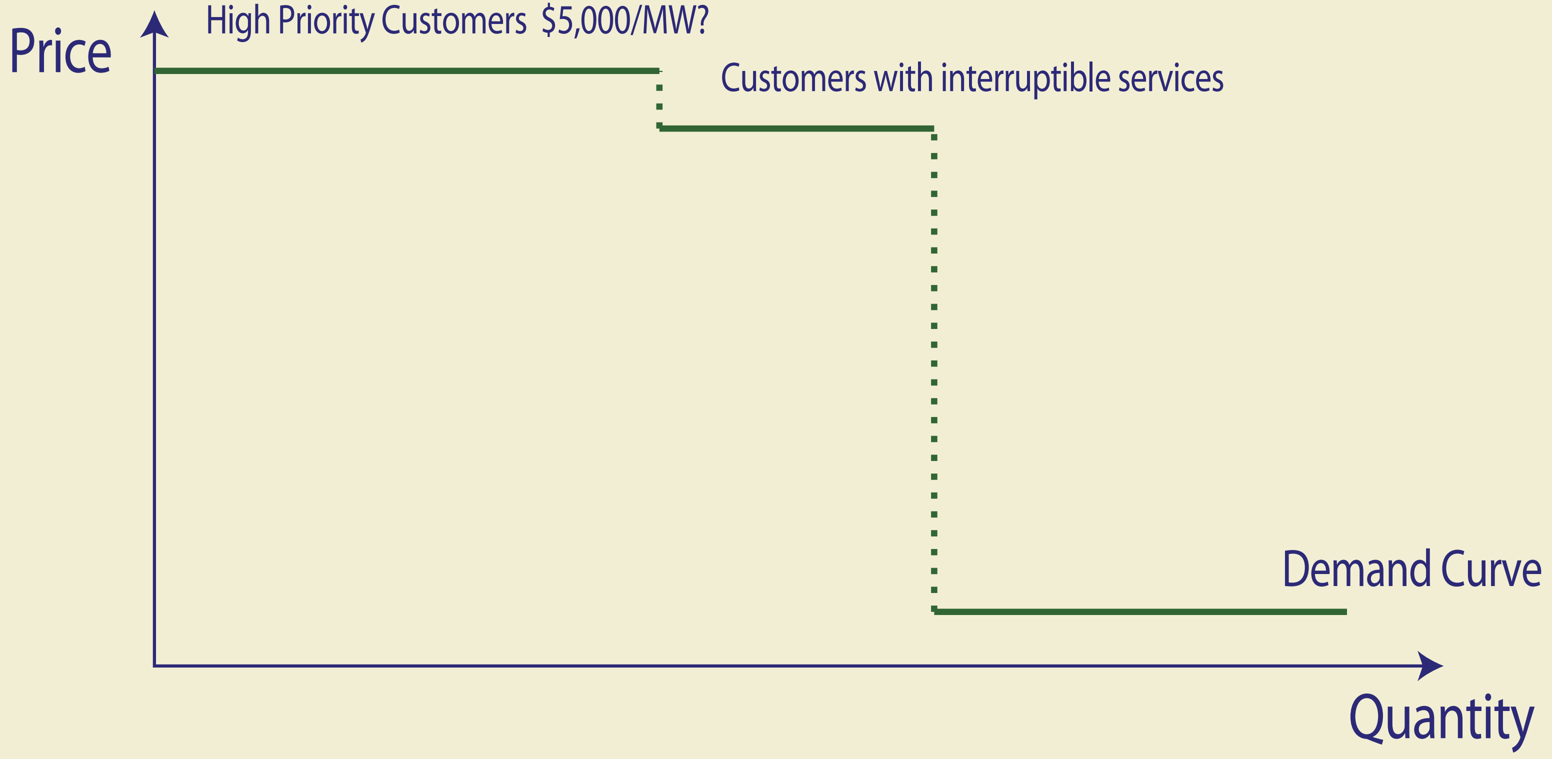
Supply & Demand

Cost of generation depends on source



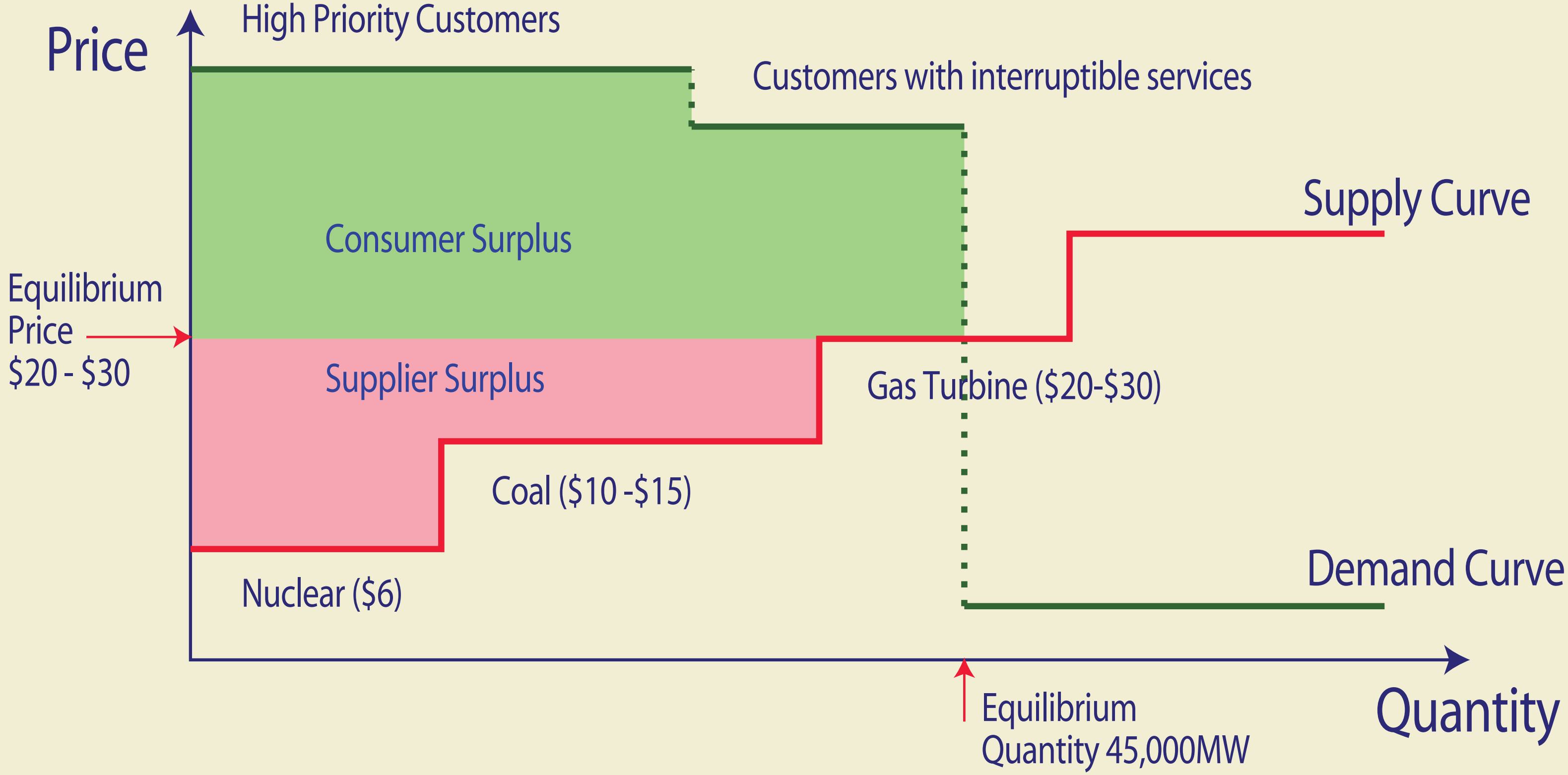
Supply & Demand

Demand for power is not flexible

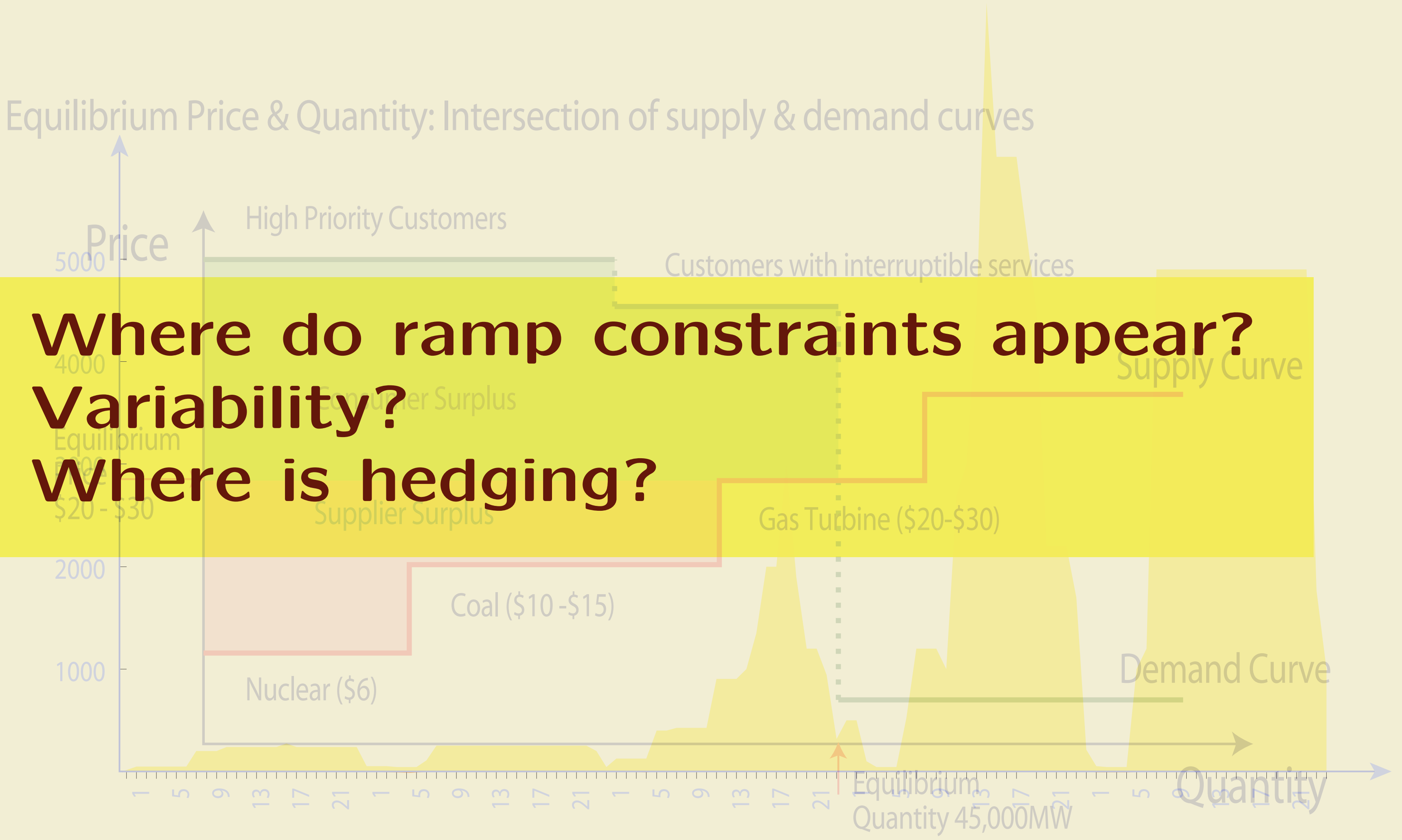


Supply & Demand

Equilibrium Price & Quantity: Intersection of supply & demand curves



Supply & Demand



Where do ramp constraints appear?

Variability?

Where is hedging?

[Main Page](#) [Market Results](#)

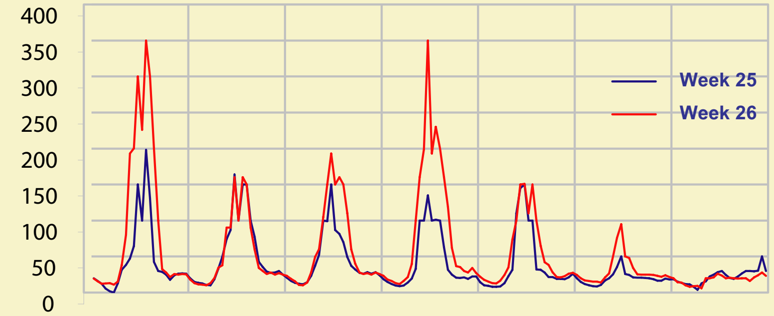
www.apx.nl

Welcome to APX!

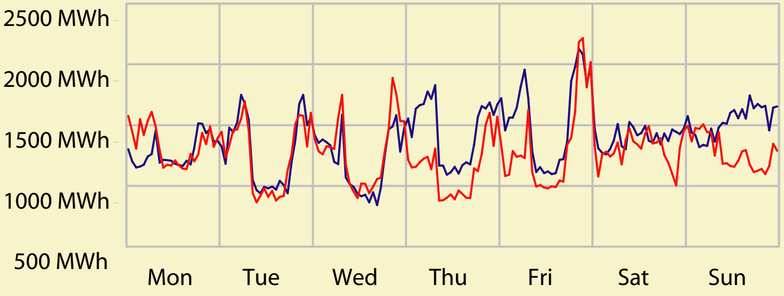
APX is the first electronic energy trading platform in continental Europe. The daily spot market has been operational since May 1999. The spot market enables distributors, producers, traders, brokers and industrial end-users to buy and sell electricity on a day-ahead basis.

The APX-index will be published daily around 12h00 (GMT +01:00) to provide transparency in the market. Prices can be used as a benchmark.

Prices (Eur/MWh)



Volumes (MWh)



Second Welfare Theorem

Each player independently optimizes ...

Consumer: value of **consumption**
minus **prices paid**
minus **disaster**

$$\mathcal{W}_{\mathrm{D}}(t) := v \min\big(D(t), G^p(t) + G^a(t)\big) - \big(p^p G^p(t) + p^a G^a(t) + c^{bo} Q_-(t)\big)$$

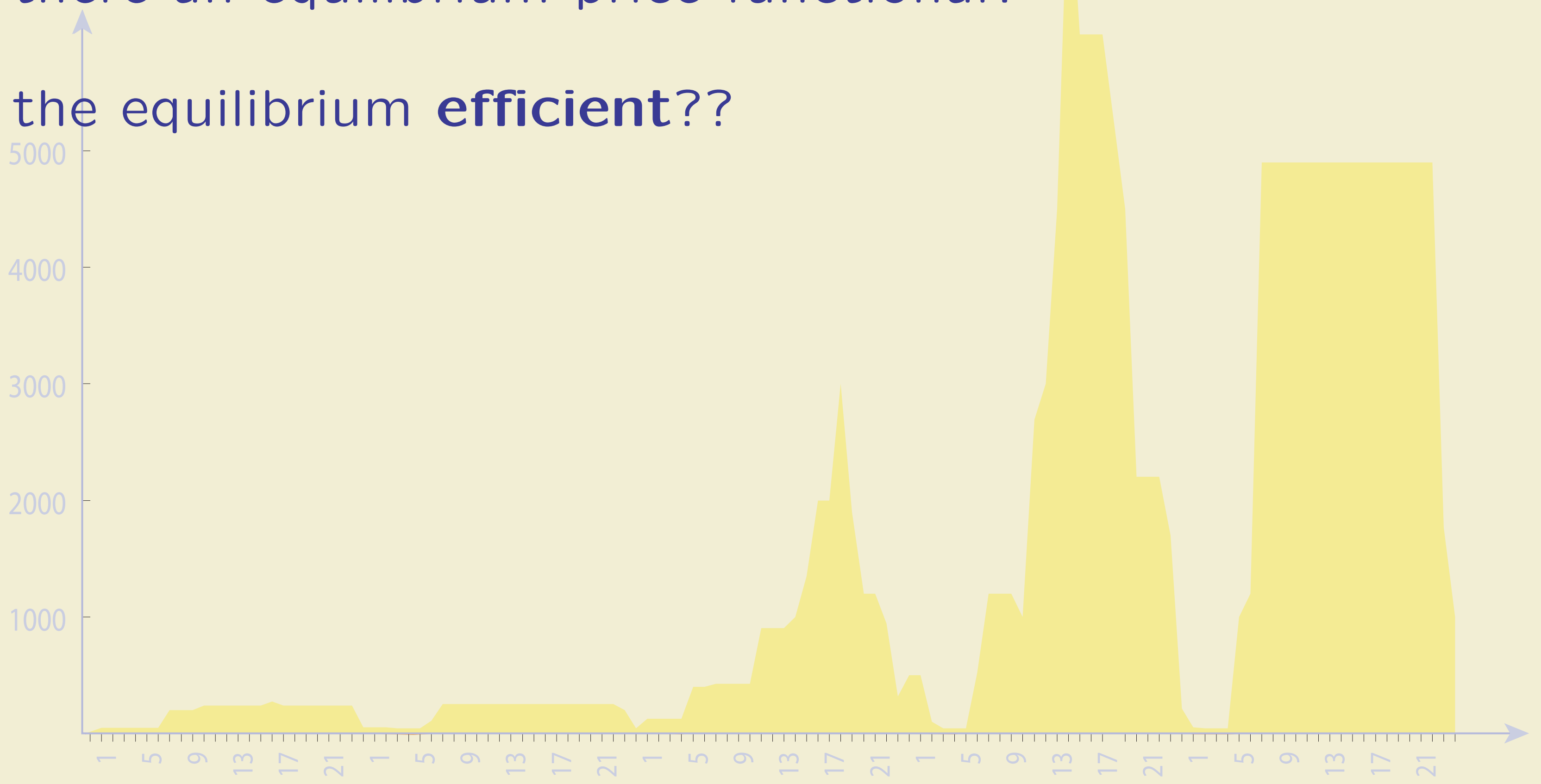
Supplier: **profits** from two sources of generation

$$\mathcal{W}_{\mathrm{S}}(t) := \big(p^p - c^p\big) G^p(t) + \big(p^a - c^a\big) G^a(t)$$

Second Welfare Theorem

Is there an equilibrium price functional?

Is the equilibrium **efficient**??

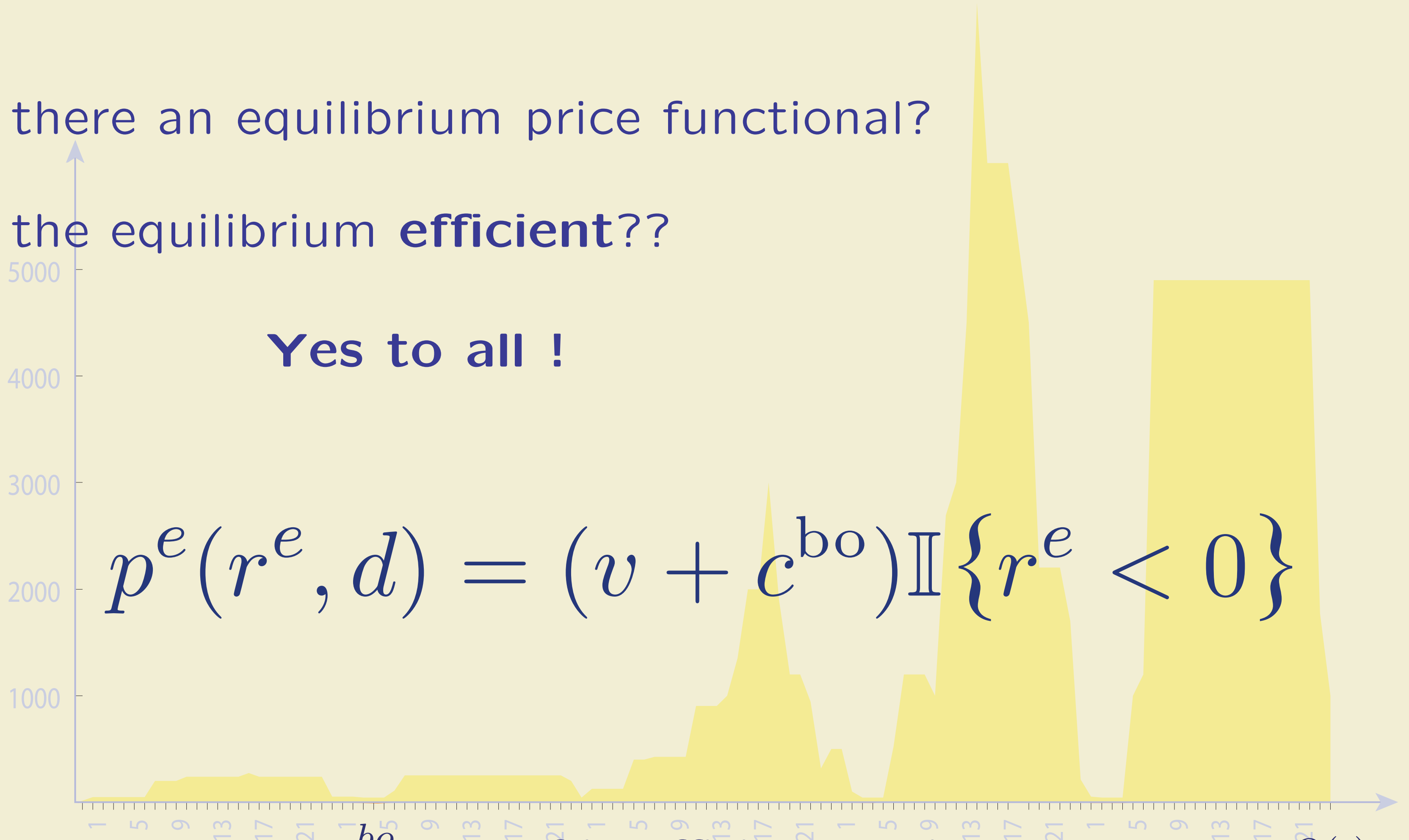


Second Welfare Theorem

Is there an equilibrium price functional?

Is the equilibrium **efficient**??

Yes to all !



$$p^e(r^e, d) = (v + c^{\mathrm{bo}})\mathbb{I}\{r^e < 0\}$$

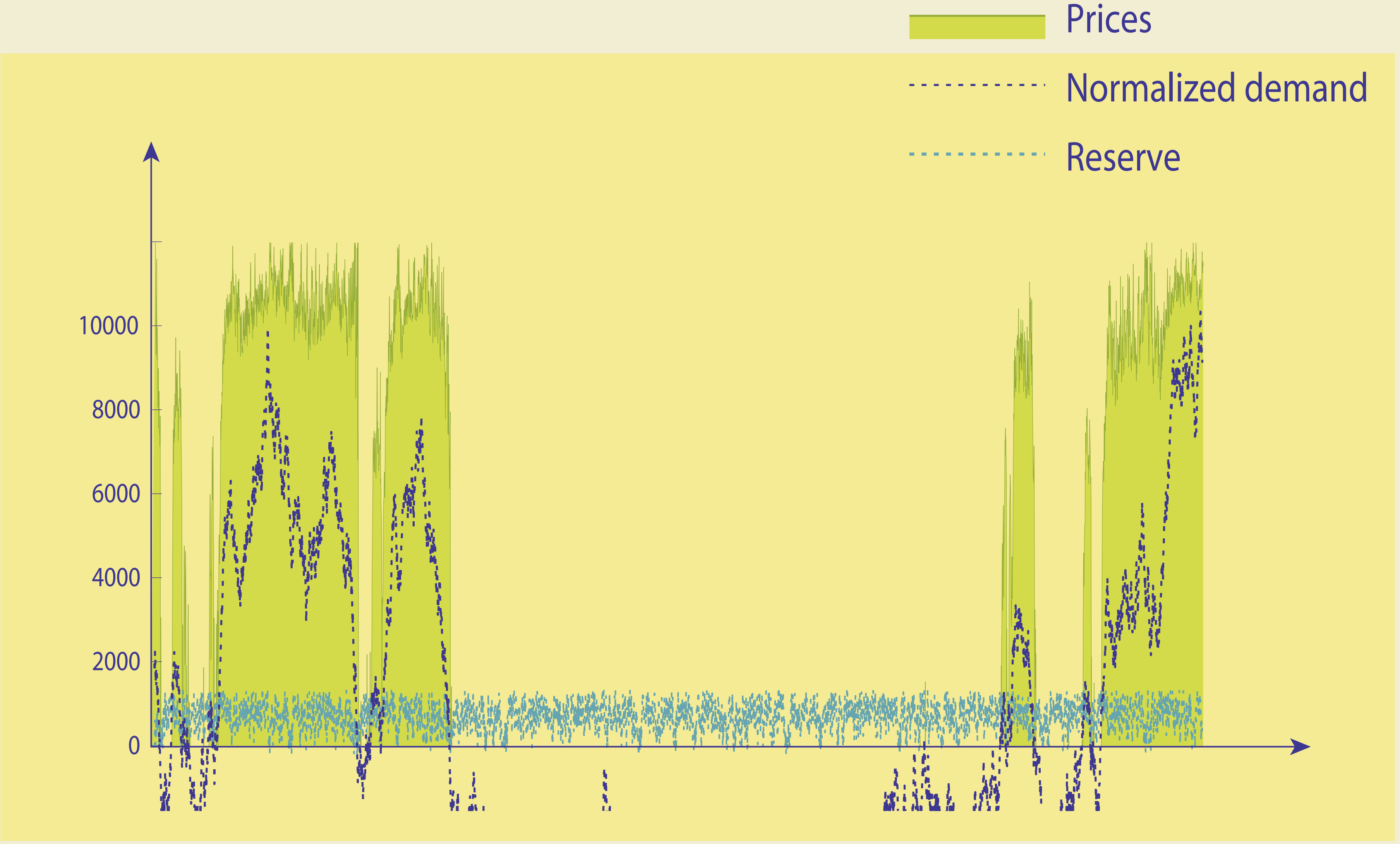
c^{bo} cost of insufficient service

v value of consumption

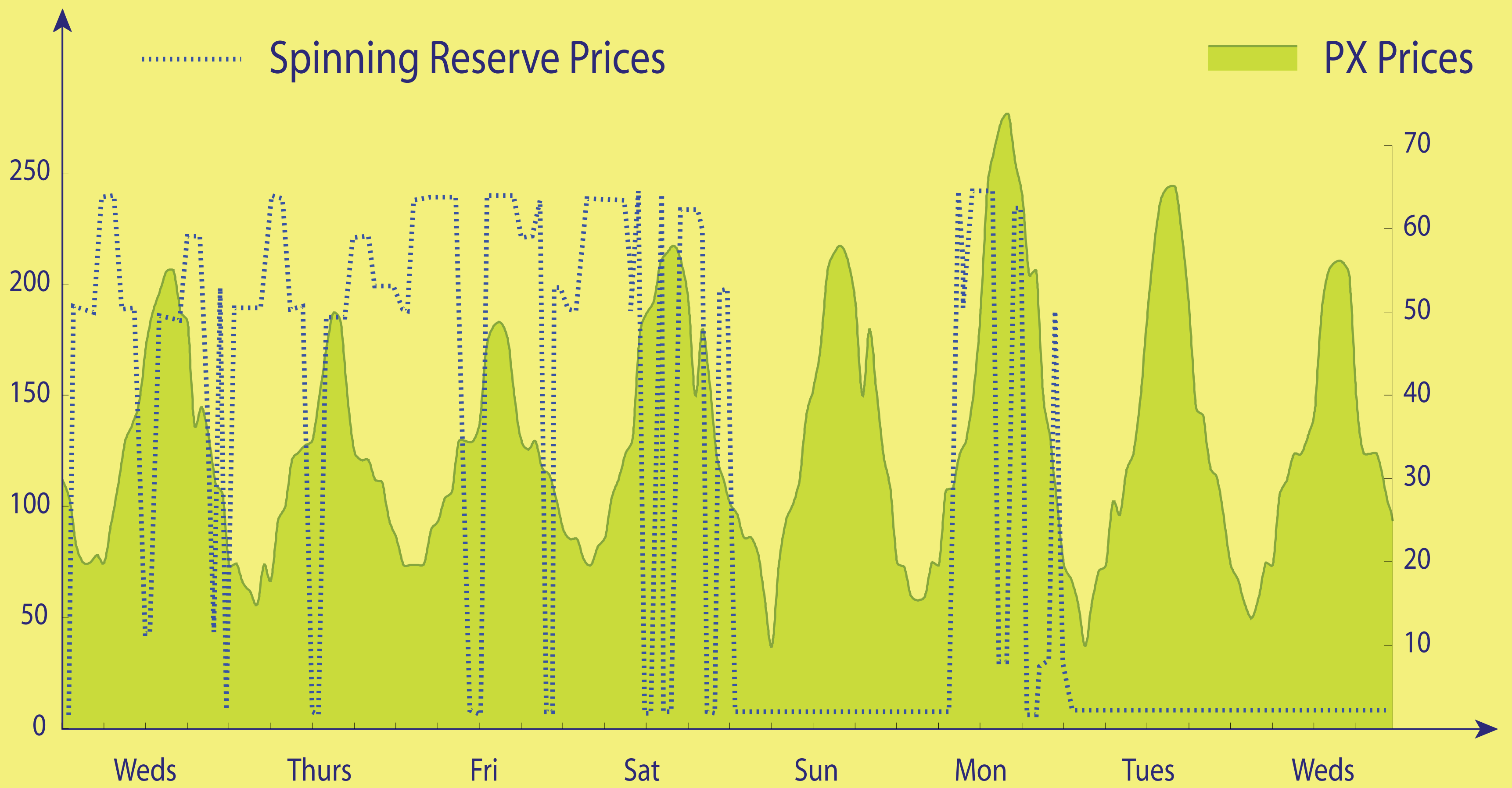
reserve $Q(t) = q$

demand $D(t) = d$

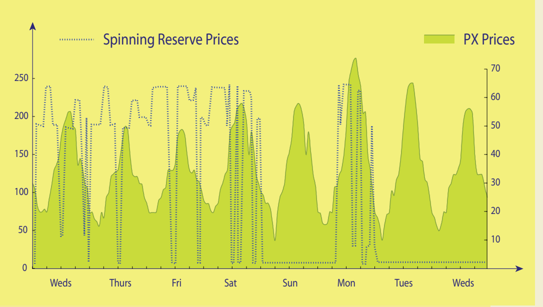
Efficient Equilibrium



Southern California, July 8-15, 1998 ...



Conclusions



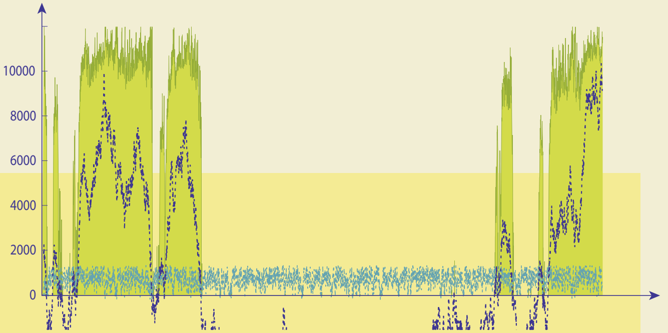
The hedging point (affine) policy
is average cost optimal

Amazing solidarity between CRW and CBM models

Deregulation is a disaster!

Future work?

Extensions and future work



Complex models:

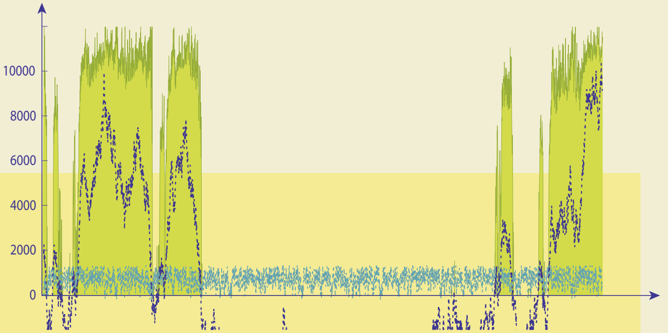
Workload or aggregate relaxations

Price caps: **No!**

Responsive demand: **Yes!**

Is ENRON off the hook: **?**

Extensions and future work



Complex models:

Workload or aggregate relaxations

Price caps: **No!**

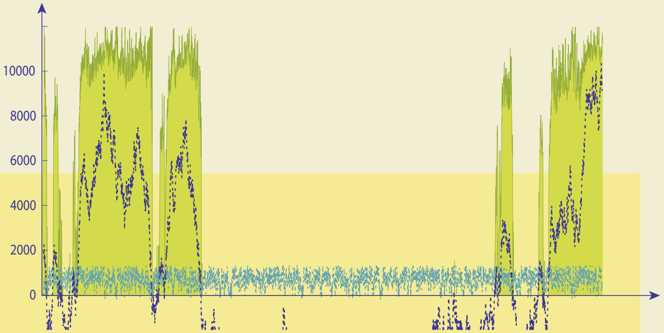
Responsive demand: **Yes!**

Is ENRON off the hook: **?**

What kind of society isn't structured on greed? The problem of social organization is how to set up an arrangement under which greed will do the least harm; capitalism is that kind of system.

-M. Friedman

Epilogue

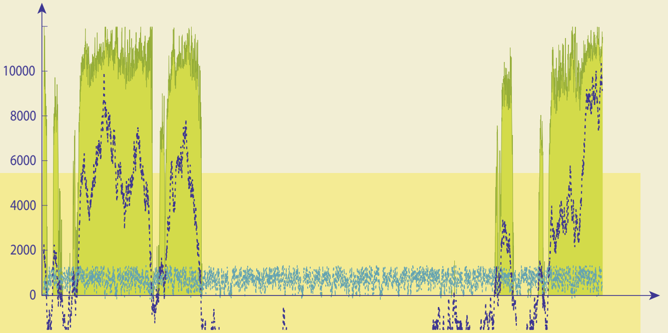


Fundamentally, there are only two ways of coordinating the economic activities of millions.
One is central direction involving the use of coercion
- *the technique of the army and of the modern totalitarian state*.

The other is voluntary cooperation of individuals
- *the technique of the marketplace*.

-Milton Friedman

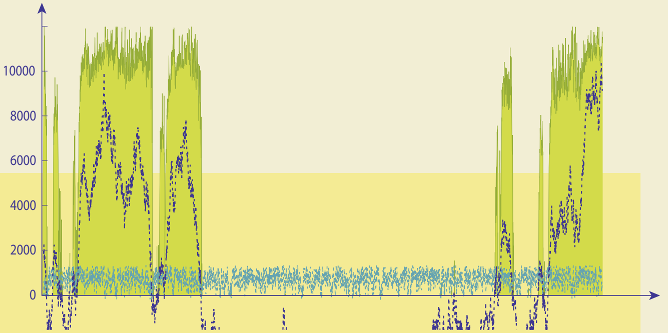
Epilogue



Justification:

1. Economic systems are complex
2. Regulators cannot be trusted

Epilogue



Justification: 1. Economic systems are complex
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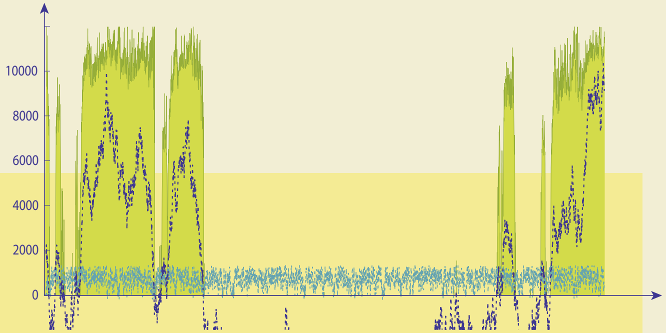
Airplanes, highway networks, cell phones... all complex

Epilogue



Justification: 1. Economic systems are complex
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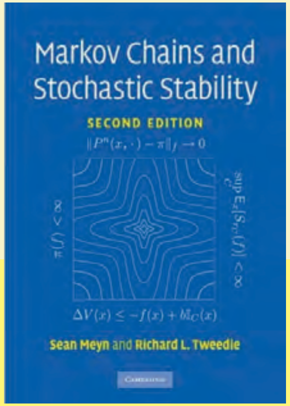
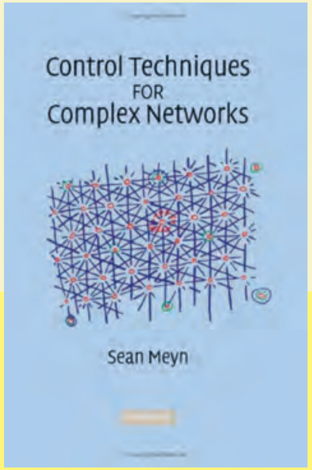
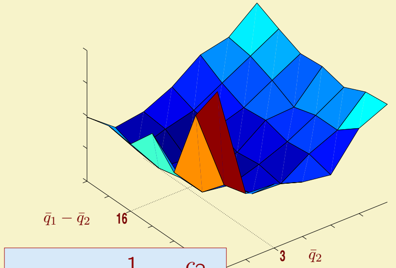
Airplanes, highway networks, cell phones... all complex

Complexity is only inherent in the uncontrolled system: In each of these examples, the *behavior of the closed loop system is very simple, provided appropriate rules of use, and appropriate feeback mechanisms are adopted*.

References







$$\bar{q}_1^* - \bar{q}_2^* = \frac{1}{\gamma_1} \log \frac{c_2}{c_1}$$

$$\bar{q}_2^* = \frac{1}{\gamma_0} \log \frac{c_3}{c_2}$$

- M. Chen, I.-K. Cho, and S. Meyn. Reliability by design in a distributed power transmission network. Automatica 2006 (invited)

- I.-K. Cho and S. P. Meyn. The dynamics of the ancillary service prices in power networks. 42nd IEEE Conference on Decision and Control. December 2003

- I.-K. Cho and S. P. Meyn. Efficiency and marginal cost pricing in dynamic competitive markets. Under revision for J. Theo. Economics. 46th IEEE Conference on Decision and Control 2006

- P. Ruiz. Reserve Valuation in Electric Power Systems. PhD dissertation, ECE UIUC 2008

Poisson's Equation

First reflection times,

$$\tau_p := \inf\{t \ge 0 : Q(t) = \bar{q}^p\}, \quad \tau_a := \inf\{t \ge 0 : Q(t) \ge \bar{q}^a\}$$

$$h(x) = \mathsf{E}_x\Big[\int_0^{\tau_p} \big(c(X(s)) - \phi\big)\, ds\Big]$$

return

Poisson's Equation

First reflection times,

$$\tau_p := \inf\{t \geq 0 : Q(t) = \bar{q}^p\}, \quad \tau_a := \inf\{t \geq 0 : Q(t) \geq \bar{q}^a\}$$

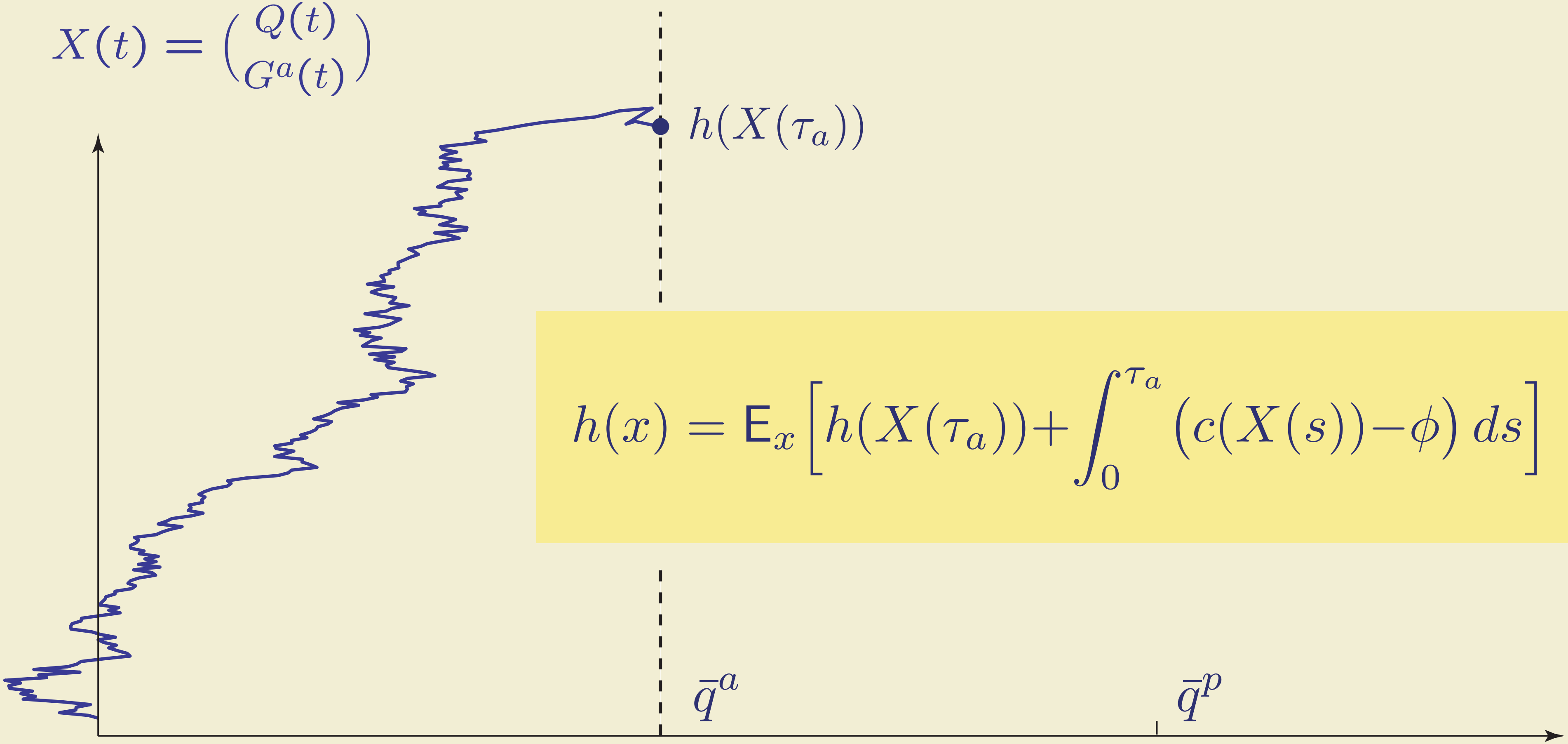
$$h(x) = \mathsf{E}_x \Big[\int_0^{\tau_p} \big(c(X(s)) - \phi \big) \, ds \Big]$$

Solves martingale problem,

$$M(t) = h(X(t)) + \int_0^t \big(c(X(s)) - \phi \big) \, ds$$

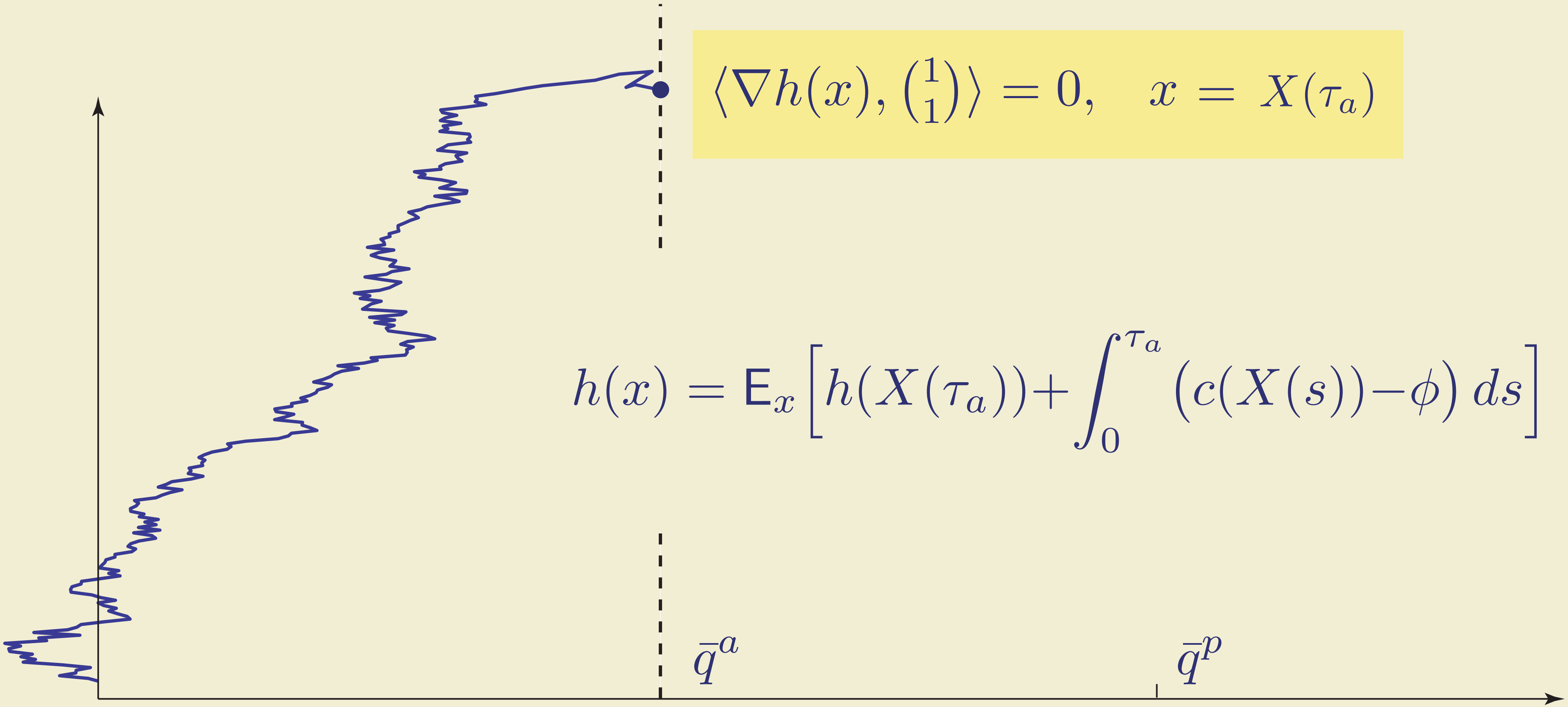
Poisson's Equation

$$X(t) = \binom{Q(t)}{G^a(t)}$$



$$h(x) = \mathsf{E}_x\Big[h(X(\tau_a)) + \int_0^{\tau_a}\big(c(X(s)) - \phi\big)\,ds\Big]$$

Derivative Representations



$$\left\langle \nabla h(x), \binom{1}{1} \right\rangle = 0, \quad x = X(\tau_a)$$

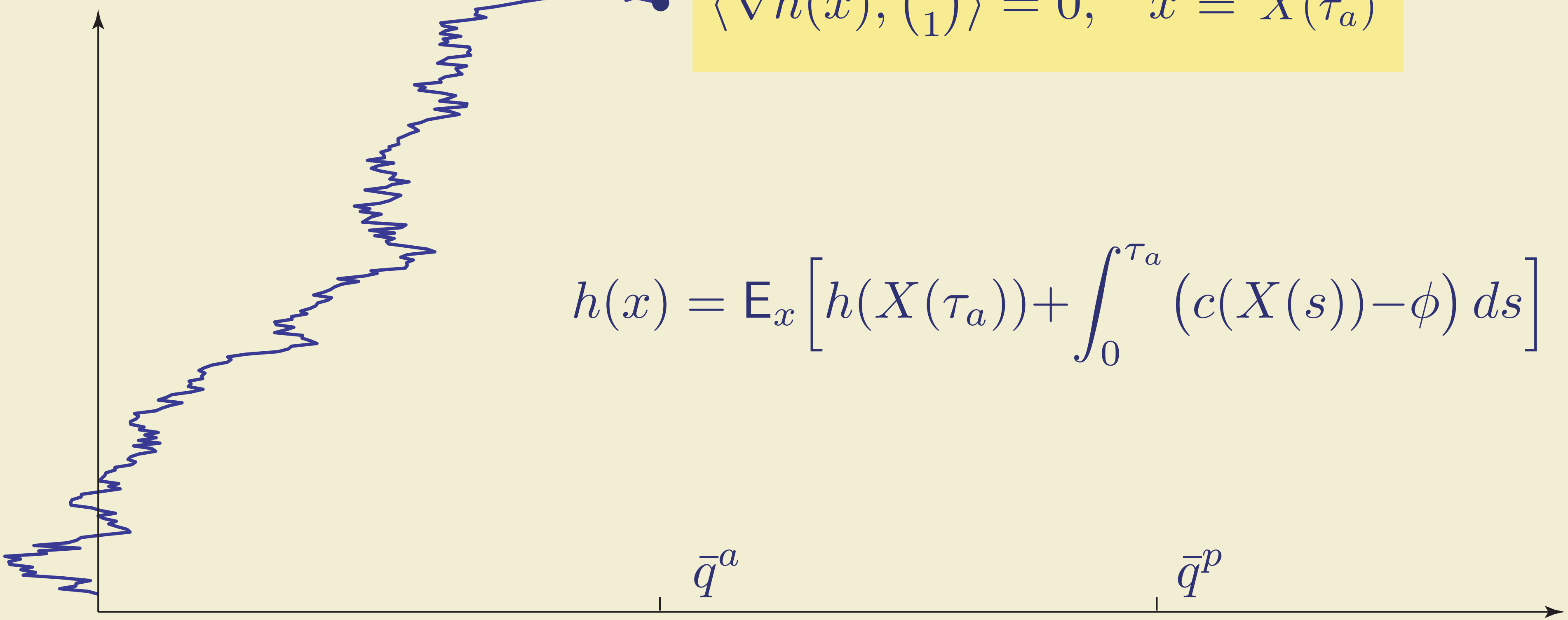
$$h(x) = \mathsf{E}_x\Big[h(X(\tau_a)) + \int_0^{\tau_a} \big(c(X(s)) - \phi\big)\, ds\Big]$$

Derivative Representations

$$\lambda_a(x) = \langle \nabla c(x), \binom{1}{1} \rangle$$
$$= c^a - \mathbb{I}\{q \le 0\} c^{bo}$$

$$\langle \nabla h(x), \binom{1}{1} \rangle = 0, \quad x = X(\tau_a)$$

$$h(x) = \mathsf{E}_x \Big[h(X(\tau_a)) + \int_0^{\tau_a} \big(c(X(s)) - \phi\big)\, ds \Big]$$



Derivative Representations

$$\langle \nabla h(x), \binom{1}{1} \rangle = \mathsf{E}_x \Big[\int_0^{\tau_a} \lambda^a(X(t))\, dt \Big]$$

$$= c^a \mathsf{E}[\tau_a] - c^{bo} \mathsf{E}_x \Big[\int_0^{\tau_a} \mathbb{I}\{Q(t) \leq 0\}\, dt \Big]$$

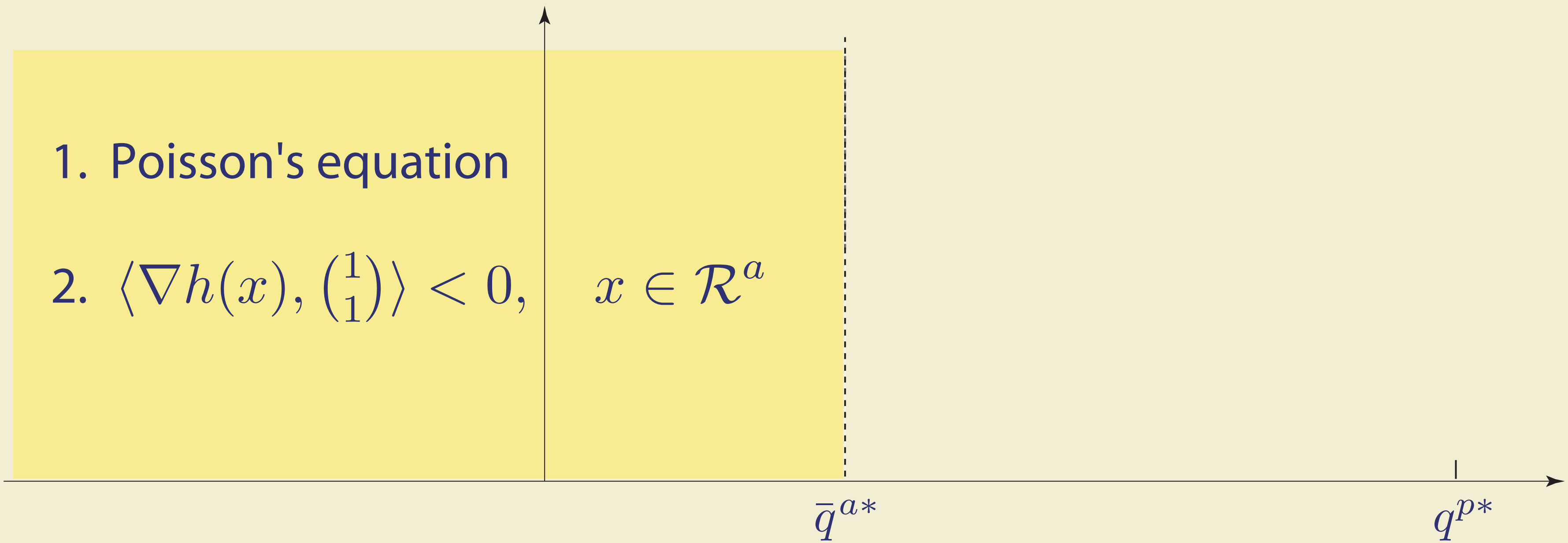
Computable based
on one-dimensional height (ladder) process,

$$H^a(t) = \bar{q}^a - Q(t)$$

Dynamic Programming Equations

If $\bar{q}^p = \bar{q}^{p*}$ and $\bar{q}^{\,a} = \bar{q}^{\,a*}$

Then h solves the dynamic programming equations,



1. Poisson's equation
2. $\langle \nabla h(x), \binom{1}{1} \rangle < 0, \quad x \in \mathcal{R}^a$
3. $\langle \nabla h(x), \binom{1}{0} \rangle < 0, \quad x \in \mathcal{R}^p$

return