Real-time Prices in an Entropic Grid

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***Abstract*— If you talk to a control theorist today about the road to achieve an efficient, reliable, and affordable electricity supply you will most likely be told that *real-time prices* will be a part of its realization. Perhaps this is true. However, we argue that appropriate design using economic models that can capture the emerging physical realities is a key requirement for achieving a reliable, and "smart" electrical grid.**

To capture the potential pitfalls of real-time prices, we present an extension of our earlier work on dynamic markets to general network settings, allowing for more general constraints on generation and transmission. We conclude in wide generality that in the economic ideal of the competitive equilibrium, the standard results follow – the equilibrium is efficient, and average prices coincide with average marginal cost. However, *these conclusions hold only on average*. More importantly, we find that in the competitive equilibrium, (a) prices can be negative, (b) prices can go well above the "choke up" price – which is the maximum price consumers are willing to pay, and (c) the variance of the price decreases with increasing demand response. We illustrate these finding through numerical experiments.

I. Introduction

The focus of the paper is on electricity markets, and in particular, a re-examination of the real-time market (RTM). Some analyses show great potential benefits of price signals to improve the reliability and efficiency of the power grid [1], [2]. We argue that the potential benefits of real-time prices may be overshadowed by their risk — risk that is evident in markets today, all over the world. We believe that these issues will become more pronounced in the future unless we pay greater attention to the impact of uncertainty and dynamics in market design.

The history of electricity markets around the world provide many examples of volatile prices, and examples of market failure. The most famous example is ENRON's role in manipulation of power prices in California in 2000-2001. Before then, unexpected price volatility brought down the Illinois market [3]. A recent example is illustrated in Figure 1. Unusually cold weather in Texas resulted in chaotic real-time prices, peaking at the price cap. Note that the price cap is $3,000, which is about 100 times the average price of $30/MWh. We know that the price cap was recently raised from $2,500, but we do not know how these limits are chosen. The state government of Texas is now attempting to determine if market manipulation played a role in disrupting energy supplies in order to influence prices [4].

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The Texas crisis is just one of many occurring in RTMs all over the world. Each crisis has its own *ex-post* explanations. Sometimes there are obvious sources. We have all heard the tape recordings that prove employees at ENRON withheld power to drive up prices. In fact, it is difficult to understand why this behavior was not anticipated prior to deregulation in California.

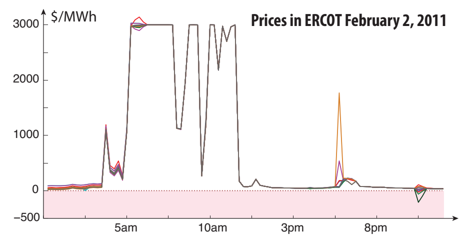

Fig. 1. Wall Street Journal, February 7, 2011, *Texas to Probe Rolling Blackouts*: Texas officials have ordered an investigation into rolling blackouts that struck the state's electric grid last week, including whether market manipulation played a role along with harsh weather in disrupting natural-gas and electricity supplies to millions of people [4].

We don't know why wild price swings are observed in Ontario, Illinois, and Australia, and in the case of Texas, there is some consensus that we will never know. In an editorial from the Feb 12, 2011 Houston Chronicle [5], the author writes, "*There will be multiple autopsies of the causes for the latest power breakdowns ... Who profited off this near-meltdown and what can be done to incentivize power producers to maintain adequate reserve capacity for emergencies rather than waiting for emergency windfalls?*".

Recent research suggests that we should not be so quick to blame the electricity suppliers. There are in fact structural explanations for RTM failure, even under ideal conditions. One of the first works to make this point is [6]. This result and its subsequent extensions are surveyed in sections II and III. It is found that with physical constraints and uncertainty taken into account, RTM prices are inherently volatile in the competitive equilibrium. This is true in the most idealized setting of a fully competitive free-market for electricity, with perfect symmetry of information, and "price-taking" players.

However, the real world is not so ideal. We cannot provide perfect statistical models of a power grid, and it is foolish to pretend that strategic behavior does not exist in today's market. That is why there is so much uncertainty about the recent events in Texas. Recent work shows that volatility and uncertainty can have tremendous impact on system operations and market outcomes [6]–[12].

The uncertainty will only grow in the future! It is hoped that our dependency on fossil fuels will diminish. Even then, the future of nuclear energy continues to be uncertain, possibly more so than before. It is clear that we must create more sources of renewable energy, such as from wind, sun, and waves.

One challenge faced today is that wind and other renewable energy resources may bring additional volatility to the grid. For this reason, we believe that many of the difficulties observed in today's RTM might increase without proper design. For example, the impact of volatile resources is studied in [10]. Using the typical policy under which all wind generation is dispatched, the volatility of wind creates a much more volatile effective load that must be served by other generators. Greater volatility requires increased reserves, thus reducing the value of these volatile resources.

It is likely that there will be solutions to these problems. With sufficient storage we can reduce the impact of volatility. It is also hoped that smart meters will reduce the negative impacts of volatility. However, again, this is a matter of design. Greater participation could actually increase volatility: the behavior of the consumers and suppliers of electricity are a source of uncertainty that is potentially greater than the wind.

We adopt the term *Entropic Grid* to describe the new power grid, with all of its new complexity and uncertainty [12]. Our intention is to highlight the fact that although new technologies and policies bring many new opportunities for a more efficient future, they also introduce the potential for higher entropy, and all of its consequences. For all the reasons mentioned above, new tools for both operations and long-term planning emerge as critical requirements for reliability in tomorrow's grid [12]–[15].

In this paper, we highlight the care required in the design of "price signals" by taking a deeper look at RTM prices. We analyze competitive equilibrium in a dynamic and uncertain market setting using the framework proposed in [9]. Our analysis reinforces the findings of [6] using a much more direct construction based on Lagrangian decomposition and duality concepts. As in [6], we find that average equilibrium prices coincide with the marginal cost, as would be expected from classical economic treatments. And, as found in [6] for a special case, the sample paths of prices in this equilibrium may never equal marginal cost. We further show that, subject to transmission constraints, equilibrium prices may become negative or they may exceed beyond the "choke-up" price that was predicted in [6]. We note that, in any realistic example today, the choke up price is well beyond any imaginable *price cap*. The largest in the world today is 10,000 Australian Dollar/MWh in Australia. A look at Figure 1 shows that the cap in ERCOT was \$3,000 on February 2nd of this year.

The paper also contains an analysis of prices for the fast responding, expensive ancillary services introduced in [8]. In [8], the solution to the "social planner's problem" was obtained, but a strategic analysis was not considered. We illustrate through numerical experiments that volatile price patterns result in very large variance of price. Given the high variance of prices, we wonder how the suppliers of fast responding, expensive ancillary services can stay in business? We wonder if there would be any incentive to enter the RTM? A potential solution to this problem is demand response from consumers with lower "disutility of blackout" (see [11] for details). We present numerical simulations to show that variance may be reduced dramatically if it is possible to shed load from responsive consumers. But the true potential of such demand response from consumers remains to be investigated.

In general, we argue that with sufficient variability, *there can be no economic outcome*! High risk and low average prices may drive generators out of business, as is feared even today in many electricity markets. Moreover, in a real world different from the ideal of the competitive equilibrium, insufficient demand flexibility may create incentive to withhold power to increase prices.

The remainder of the paper is organized as follows. Section II surveys market models and economic analysis from [6], [9]. In Section III, we extend these results to obtain properties of equilibrium prices in the (efficient) competitive equilibrium. Examples illustrating these results are provided in Section IV with impacts of transmission constraints explored in Section IV-A and the price impacts for ancillary service discussed in Section IV-B. Concluding remarks and several questions for future research are contained in Section V.

II. Dynamic Competitive Equilibrium

In this section, we review the models and results from [6]–[10]. The market analyses contained in these papers have been formulated under the framework of dynamic general equilibrium assuming *price-taking* behavior, thereby eliminating market manipulation. That is, it is assumed that no market player is large enough to influence prices. This is clearly an idealization of any market operating in the world today.

Efficiency of the competitive equilibrium is defined with respect to the outcome of the social planner's problem. This is the solution to a centralized optimal control problem representing generation reserve requirements across the grid. The reserve management approaches described in [7], [16] are based on a version of the models described in this section.

A. Dynamic dispatch for operations

The power grid is represented by a graph in which each node represents a bus, and each link represents a transmission line. There are ℓ_n nodes, denoted $\mathcal{N} = \{1, \dots, \ell_n\}$, and L transmission lines, indexed by $\{1, 2, ..., L\}$. The network is assumed to be connected. A lossless DC model is used to characterize the relationship between generation, demand, and power on the various links. For simplicity, in this paper we assume that at each node there is exactly one source of generation, and one exogenous demand.

Demand-side: We denote by $D_n(t)$ the demand at time t, at bus n, and by $E_{\text{D}n}(t)$ the energy withdrawn by the consumer at that bus. We assume that there is no free disposal for energy, which requires that $E_{\text{D}n}(t) \leq D_n(t)$ for all t. If sufficient generation is available at bus n at time t, then $E_{\mathrm{D}n}(t) = D_n(t)$. In the event of insufficient generation, we have $E_{\mathrm{D}n}(t) < D_n(t)$, i.e., the consumer experiences blackout.

The consumer obtains value on consuming energy and disutility for not meeting demand. These are represented by possibly nonlinear functions,

$$\begin{aligned} \text{Utility of consumption:} \qquad & v_n(E_{\mathrm{D}n}(t))\,, && \text{(II.1a)} \\ \text{Disutility of blackout:} \qquad & c_n^{\mathrm{bo}}(D_n(t) - E_{\mathrm{D}n}(t))\,. && \text{(II.1b)} \end{aligned}$$

Each of these cost functions is assumed to be continuously differentiable in the analysis that follows; in our examples we consider piecewise linear functions for purposes of computation. The consumer must pay for energy at price $P_n(t)$.[1] The welfare of the consumer at time t is the signed sum of his benefits and costs:

$$\begin{aligned} \mathcal{W}_{\mathrm{D}}(t) := \sum_n \big[v_n(E_{\mathrm{D}n}(t)) \; - \; & c_n^{\mathrm{bo}}(D_n(t) - E_{\mathrm{D}n}(t)) \\ & - \; P_n(t) E_{\mathrm{D}n}(t) \big]\,. \end{aligned} \tag{II.2}$$

We use $D(t)$, $E_{\mathrm{D}}(t)$, and $P(t)$ to denote the associated N-dimensional column vectors, and we use bold face font to denote the entire sample path. For instance, $\boldsymbol{P} := \{P(t) : t \geq 0\}$.

Supply-side: We denote by $E_{\mathrm{S}n}(t)$ and $R_{\mathrm{S}n}(t)$ the energy and reserve produced by the supplier at bus n. Available generation capacity, $\boldsymbol{G}_{\mathrm{S}}$, coincides with $\boldsymbol{E}_{\mathrm{S}} + \boldsymbol{R}_{\mathrm{S}}$.

The production cost at time t for energy injected at bus n is denoted by $c_n^{\mathrm{E}}(E_{\mathrm{S}n}(t))$, and for the reserve provided at that bus is $c_n^{\mathrm{R}}(R_{\mathrm{S}n}(t))$. The supplier receives the revenue $P_n E_{\mathrm{S}n}(t)$ for producing energy. The welfare of the supplier at time t is the difference between his revenue and the costs,

$$\mathcal{W}_{\mathrm{S}}(t) := \sum_n \big[P_n E_{\mathrm{S}n}(t) - c_n^{\mathrm{E}}(E_{\mathrm{S}n}(t)) - c_n^{\mathrm{R}}(R_{\mathrm{S}n}(t)) \big]\,. \tag{II.3}$$

Constraints: The remaining assumptions on the dynamic network model are described as follows:

(i) Generation capacity is *rate constrained*: For all $t_1 > t_0 \geq 0$,

$$\zeta_n^- \leq \frac{E_{\mathrm{S}n}(t_1) - E_{\mathrm{S}n}(t_0)}{t_1 - t_0} + \frac{R_{\mathrm{S}n}(t_1) - R_{\mathrm{S}n}(t_0)}{t_1 - t_0} \leq \zeta_n^+\,. \tag{II.4}$$

(ii) Lossless network, so it neither generates nor consumes energy. Consequently, the network is subject to the supply-demand balance constraint,

$$1^T E_{\mathrm{S}}(t) = 1^T E_{\mathrm{D}}(t)\,, \quad t \geq 0\,. \tag{II.5}$$

(iii) Power flows over the network are consistent with the DC power flow model. Suppose bus 1 is selected as the reference bus, based on which the *injection shift factor matrix* $H \in [-1,1]^{N\times L}$ is defined. Note that H_{nl} represents the power that is distributed on line l when 1 MW is injected into bus n and withdrawn at the reference bus. If $f_l^{\max}$ denotes the capacity of transmission line l and $H_l \in \mathbb{R}^N$ represents the l-th column of H, the capacity constraint for line l can be expressed as,

$$-\,f_l^{\max} \leq (E_{\mathrm{S}} - E_{\mathrm{D}})^T H_l \leq f_l^{\max}\,. \tag{II.6}$$

We summarize all of these constraints by writing,

$$(\boldsymbol{E}_{\mathrm{S}}, \boldsymbol{R}_{\mathrm{S}}, \boldsymbol{E}_{\mathrm{D}}) \in \mathsf{X}\,. \tag{II.7}$$

The network shown in Figure 2 – which we refer to as the *Texas model* – provides an example of the general model in which there are sources of demand and supply at each of three nodes. If the impedances are identical in the three transmission lines, then with bus 1 chosen as the reference bus, the injection shift factor matrix is given by

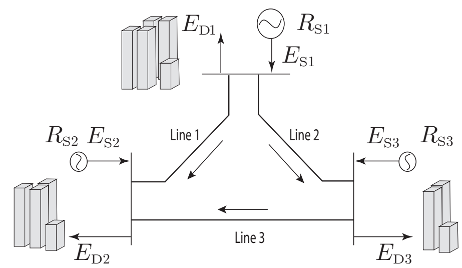

Fig. 2. Texas model: a simplified 3-bus network.

$$H = \tfrac{1}{3} \begin{bmatrix} 0 & 0 & 0 \\ -2 & -1 & -1 \\ -1 & -2 & 1 \end{bmatrix}\,. \tag{II.8}$$

The three dimensional vector of line flows, F, is given by

$$F = [E_{\mathrm{S}\mathit{1}} - E_{\mathrm{D}\mathit{1}},\ E_{\mathrm{S}\mathit{2}} - E_{\mathrm{D}\mathit{2}},\ E_{\mathrm{S}\mathit{3}} - E_{\mathrm{D}\mathit{3}}]H,$$

where the directions of positive power flows are as indicated by arrows in the figure.

B. Dynamic competitive equilibrium for dynamics markets

In typical analyses of static market models, the competitive equilibrium prices are equal to the marginal production costs. In the example considered in [6], we find that this holds only on average: The sample path behavior of prices can look as erratic as the worst days during the crises in Illinois or California in the 1990s, or in Texas and Australia today. Here, we show how these conclusions can be obtained via Lagrangian relaxation techniques presented in [9].

To capture the impact of network constraints and exploit network structure we introduce a third player – the network. This is motivated in part by current practice: The transmission grid is operated by a third entity separate from consumers/suppliers in every electricity market in the world today. We find it convenient to introduce a "network welfare function" to define a competitive equilibrium for the power grid market model. The welfare of the network at time t represents the "toll charges" for the transmission of energy. At time t, this is defined by,

$$\mathcal{W}_{\mathrm{T}}(t) := \sum_n \big[P_n(E_{\mathrm{D}n}(t) - E_{\mathrm{S}n}(t)) \big]\,. \tag{II.9}$$

To evaluate the welfare performance of the market, we introduce into our analysis a *social planner* who aims

[1] Observe that prices are determined by location. In the language of today's markets, they are *locational prices*.

to maximize the economic well-being of everyone in the system. The social planner uses the total welfare, denoted by $\mathcal{W}_{\text{tot}}(t)$, to measure the economic well-being of the system with

$$\mathcal{W}_{\text{tot}}(t) := \mathcal{W}_{\text{S}}(t) + \mathcal{W}_{\text{D}}(t) + \mathcal{W}_{\text{T}}(t)\,. \tag{II.10}$$

The total welfare is in fact the sum of $\{v_n(E_{\text{D}n}(t)) - c_n^{\text{bo}}(D_n(t) - E_{\text{D}n}(t)) - c_n^{\text{E}}(E_{\text{S}n}(t)) - c_n^{\text{R}}(R_{\text{S}n}(t))\}$ over all nodes n. Note that it is independent of the price process.

Definition 2.1: The social planner's problem (SPP) is

$$\max_{\boldsymbol{E}_{\text{D}},\boldsymbol{E}_{\text{S}},\boldsymbol{R}_{\text{S}}} \int e^{-\gamma t}\mathcal{W}_{\text{tot}}(t)\,dt\,. \tag{II.11}$$

subject to the operational/physical constraint (II.7). Its solution, if it exists, is called an *efficient allocation*.

We now impose several idealistic assumptions for this dynamic model, each of which is an extension of what is assumed in the competitive equilibrium analysis of a static model.

(i) Consumers and suppliers share equal information. This is modeled as a *filtration*: an increasing family of σ-algebras, denoted $\mathcal{H} = \{\mathcal{H}_t : t \geq 0\}$. The demand process, and the decisions of the consumers and the suppliers are adapted to this filtration.

(ii) There is a price process vector $\boldsymbol{P}^e$ that is adapted to $\{\mathcal{H}_t : t \geq 0\}$. However, prices are *exogenous*: for each $t_0 > 0$, the future prices $\{P^e(t) : t > t_0\}$ are conditionally independent of $\{E_{\text{D}}(t), E_{\text{S}}(t) : t \leq t_0\}$, given current and past prices $\{P^e(t) : t \leq t_0\}$.

Assumption (ii) is known as the *price-taking assumption*

The Lagrangian decomposition of (2.1) is obtained on relaxing the supply-demand balanced and the network constraints. For two stochastic processes $\boldsymbol{F}$ and $\boldsymbol{G}$, each adapted to $\mathcal{H}_t$, we denote $\langle \boldsymbol{F}, \boldsymbol{G}\rangle := \mathsf{E}\Big[\int e^{-\gamma t} F(t)G(t)\,dt\Big]$. Let $\lambda(t)$, $\mu_l^+(t)$ and $\mu_l^-(t)$ denote stochastic processes adapted to $\mathcal{H}$,and consider the Lagrangian relaxation.

Definition 2.2: The Lagrangian of the SPP is

$$\begin{aligned}\mathcal{L} = &-\langle \mathcal{W}_{\text{tot}}, \mathbf{1}\rangle + \langle \boldsymbol{\lambda}, (1\cdot \boldsymbol{E}_{\text{D}} - 1\cdot \boldsymbol{E}_{\text{S}})\rangle \\ &+ \sum_l \langle \boldsymbol{\mu}_l^+, (\boldsymbol{E}_{\text{S}} - \boldsymbol{E}_{\text{D}})\cdot H_l - f_l^{\max}\rangle \\ &+ \sum_l \langle \boldsymbol{\mu}_l^-, -(\boldsymbol{E}_{\text{S}} - \boldsymbol{E}_{\text{D}})\cdot H_l - f_l^{\max}\rangle\,,\end{aligned}$$

where $\mu_l^+(t) \geq 0$ and $\mu_l^-(t) \geq 0$ for all t and l.

The dual functional Φ is defined as the supremum of the Lagrangian over all adapted processes $\boldsymbol{E}_{\text{S}}$, $\boldsymbol{E}_{\text{D}}$, and $\boldsymbol{R}_{\text{S}}$. The supremum decomposes into three problems: one for the consumer, one for the supplier, and one for the transmission operator as follows, note the term for transmission yields constant value,

$$\begin{aligned}\Phi = &\max_{\boldsymbol{E}_{\text{D}n}} \sum_n \{\langle v_n(\boldsymbol{E}_{\text{D}n}) - c_n^{\text{bo}}(\boldsymbol{D}_n - \boldsymbol{E}_{\text{D}n}), \mathbf{1}\rangle - \langle \boldsymbol{P}_n, \boldsymbol{E}_{\text{D}n}\rangle\} \\ &+ \max_{\boldsymbol{E}_{\text{S}n},\boldsymbol{R}_{\text{S}n}} \sum_n \{\langle \boldsymbol{P}_n, \boldsymbol{E}_{\text{S}n}\rangle - \langle c_n^{\text{E}}(\boldsymbol{E}_{\text{S}n}) + c_n^{\text{R}}(\boldsymbol{R}_{\text{S}n}), 1\rangle\} \\ &+ \sum_l \langle \boldsymbol{\mu}_l^+ + \boldsymbol{\mu}_l^-, f_l^{\max}\rangle\,,\end{aligned}$$

where $\boldsymbol{P}_n = \boldsymbol{\lambda} + \sum_l (\boldsymbol{\mu}_l^- - \boldsymbol{\mu}_l^+)H_{ln}$. Under general conditions there is no duality gap: there are processes $\boldsymbol{\lambda}^*$, $\boldsymbol{\mu}_l^{+*}$ and $\boldsymbol{\mu}_l^{-*}$ for which the solutions to (II.12) and the SPP (II.11) coincide. Moreover, by inspection it follows that this forms a solution to the competitive equilibrium.

III. Equilibrium Prices

Because of unique features of a power market, the equilibrium price process looks very different from the solution of a static model. We revisit previous results for single node case and we provide the generalization of these results for multiple node case.

A. Single node model

With $\ell_n = 1$ in the model of Section II-A and linearity assumptions on cost and utility/disutility functions, we obtain the single-bus model used in [6]. In [6], we found that the equilibrium price process $\boldsymbol{P}^e$ is given by,

$$P^e(t) = (v + c^{\text{bo}})\mathbb{I}\{R^*(t) < 0\}, \tag{III.12}$$

where $\boldsymbol{R}^*$ is the reserve process in the solution of SPP. The quantity $v + c^{\text{bo}}$ is known as the *choke-up price* since it is the maximum the consumer is willing to pay (based on a static analysis). Since the choke-up price can be extremely large in a real power system, equilibrium prices show tremendous volatility. However, in this prior work it was shown that the *average* price coincides with marginal cost c for generation, in the sense that

$$\gamma \mathsf{E}\Big[\int e^{-\gamma t}P^e(t)\,dt\Big] = c\,. \tag{III.13}$$

This holds only when initial reserves are sufficiently large. The derivation was by direct calculation.

In this section we show that the same conclusions can be derived for the general model using Lagrange multiplier techniques. The conclusions are slightly different because of a deviation from the model of [6]. The generation $G(t) := E(t) + R(t)$ was assumed to be constrained by ramp rate in [6], but no other constraints were imposed. In particular, negative values were allowed since the generation $G(t)$ was assumed to be normalized (the deviation from the power allocation determined in the day-ahead market (DAM)).

For simplicity, we assume here that $\mathsf{X}_{\text{S}}^{\circ}$ is defined as in [9] by the ramp constraints (II.4), and subject to the non-negativity constraints $E_{\text{S}}(t) \geq 0$, $R_{\text{S}}(t) \geq 0$, for all t.

We first establish the formula for $\boldsymbol{P}^e$.

Proposition 3.1: Suppose that $(\boldsymbol{E}^*, \boldsymbol{R}^*)$ is a solution to the SPP that defines a competitive equilibrium with price process $\boldsymbol{P}^e$. Then,

$$P^e(t) = \nabla v\,(E^*(t)) + \nabla c^{\text{bo}}\,(D(t) - E^*(t)),\ \ t \geq 0. \tag{III.14}$$

Proof. In the single bus model we have $\mathcal{W}_{\text{D}}(t) := v(E_{\text{D}}(t)) - c^{\text{bo}}(D(t) - E_{\text{D}}(t)) - P^e(t)E_{\text{D}}(t)$. The formula follows because $\boldsymbol{E}^* = \boldsymbol{E}_{\text{D}}$ in the competitive equilibrium, and the consumer is myopic (recall that the consumer does not consider ramp constraints). ■

To find the average price we must consider the optimization problem posed by the supplier. We then consider a Lagrangian relaxation, in which the constraint $E_s(0) + R_s(0) = g_0$ is captured in the Lagrange multiplier ν. For this we define the Lagrangian,

$$\begin{aligned}\mathcal{L}_s(\boldsymbol{E}_s, \boldsymbol{R}_s, \nu) = \;& \mathsf{E}\Big[\int e^{-\gamma t}\mathcal{W}_s(t)\,dt\Big] \\ & -\nu[E_s(0) + R_s(0) - g_0]\,. \end{aligned} \tag{III.15}$$

The following result is a consequence of the local Lagrange multiplier theorem [17].

Lemma 3.2: Suppose $(\boldsymbol{E}_s, \boldsymbol{R}_s)$ maximizes $\mathcal{L}_s(\boldsymbol{E}_s, \boldsymbol{R}_s, 0)$ over pairs in $\mathsf{X}_s^{g_0}$ (the set of feasible $(\boldsymbol{E}_s, \boldsymbol{R}_s)$, subject to the given initial condition g_0). Then, there exist $\nu^* \in \mathbb{R}$ such that $(\boldsymbol{E}_s, \boldsymbol{R}_s)$ maximizes $\mathcal{L}_s(\boldsymbol{E}_s, \boldsymbol{R}_s, \nu^*)$ over the larger set of functions X_s°.

The next result is a construction required in an application of the Lagrange multiplier result Lemma 3.2.

Lemma 3.3: Suppose that $(\boldsymbol{R}_s, \boldsymbol{E}_s)$ belongs to X_s°. Then there exists a family of solutions $\{(\boldsymbol{R}_s, \boldsymbol{E}_s^\alpha) : |\alpha| \le 1\} \subset \mathsf{X}_s^\circ$ satisfying $E_s^\alpha(0) = \max(E_s(0)+\alpha, 0)$, $|E_s^\alpha(t) - E_s(t)| \le |\alpha|$ for all α, and for $t > 0$, $\lim_{\alpha\to 0} \frac{1}{\alpha}(E_s^\alpha(t) - E_s(t)) = \mathbf{1}_s^+(t) := \mathbb{I}\{E_s(t) > 0\}$.

Combining the two Lemmas we easily obtain the following extension of (III.13).

Theorem 3.4: Suppose that $(\boldsymbol{E}^*, \boldsymbol{R}^*)$ is a solution to the SPP that defines a competitive equilibrium with price process $\boldsymbol{P}^e$. Suppose that $E^*(0) > 0$. Moreover, assume that the following bounds hold: The processes $\boldsymbol{E}^*$ is square integrable, meaning that $\langle \boldsymbol{E}^*, \boldsymbol{E}^*\rangle < \infty$, and the cost function c_s satisfies $c_s(e) + |\nabla c_s(e)| \le c_0(1 + e^2)$ for some $c_0 > 0$ and all $e \ge 0$. Then the average price coincides with average marginal cost of energy plus the scaled sensitivity term ν^*:

$$\begin{aligned}&\gamma\mathsf{E}\Big[\int_0^\infty e^{-\gamma t}\mathbf{1}_s^+(t)P^e(t)\,dt\Big] \\ =&\gamma\mathsf{E}\Big[\int_0^\infty e^{-\gamma t}\mathbf{1}_s^+(t)\nabla c_s\left(E^*(t)\right)dt\Big] + \gamma\nu^*\,.\end{aligned} \tag{III.16}$$

Proof. The Lagrangian $\mathcal{L}(\boldsymbol{E}^\alpha, \boldsymbol{R}^*, \nu)$ is differentiable as a function of α under the assumptions of the theorem, and we have the expression,

$$\begin{aligned}\frac{d}{d\alpha}\mathcal{L}(\boldsymbol{E}^\alpha, \boldsymbol{R}^*, \nu^*) = \;& \mathsf{E}\Big[\int e^{-\gamma t}\frac{d}{d\alpha}\mathcal{W}_s^\alpha(t)\,dt\Big] \\ & -\frac{d}{d\alpha}\nu^*[E^\alpha(0) + R^*(0) - g_0]\,.\end{aligned} \tag{III.17}$$

where $\mathcal{W}_s^\alpha(t) := P^e(t)E_s^\alpha(t) - c_s^E(E_s^\alpha(t)) - c_R^E(R^*(t))$. In this calculation the square integrability assumption and bounds on c_s are used to justify taking the derivative under the expectation and integral in (III.15).

The conclusion of the theorem then follows from two facts. First is optimality of the Lagrangian at $\alpha = 0$, giving $\frac{d}{d\alpha}\mathcal{L}(\boldsymbol{E}^\alpha, \boldsymbol{R}^*, \nu) = 0$ for $\alpha = 0$. We then apply Lemma 3.3 which allows an application of the chain rule to obtain,

$$\frac{d}{d\alpha}\mathcal{W}_s^\alpha(t)\Big|_{\alpha=0} = \mathbf{1}_s^+(t)\big(P^e(t)E^*(t) - \nabla c_s\left(E^*(t)\right)\big)\,.$$

Evaluating (III.17) at $\alpha = 0$ then gives the desired result. ■

Thus, we can establish (III.13). Note that the average price depends on the initial value g_0, and this dependence is captured through the sensitivity term ν^*. When ramping down is unconstrained, $\nu^* \ge 0$.

B. Network constraints

We now turn to the general electricity market model subject to network constraints introduced in Section II. We introduce a fictitious market, $\mathfrak{S}$-market in our analysis.

Definition 3.5: The $\mathfrak{S}$-market is defined as follows:

(i) The consumer and transmission models are unchanged. The operational/physical constraints (II.7) on $(\boldsymbol{E}_s, \boldsymbol{R}_s)$ are relaxed.

(ii) The welfare functions of the consumer and the network are unchanged.

(iii) The welfare function of the supplier is *identically zero*. This is achieved by overriding the production cost functions as follows:

$$c_n^{E\mathfrak{S}}(E_{sn}(t)) = P_n^e(t)E_{sn}(t)\,,\quad c_n^{R\mathfrak{S}}(R_{sn}(t)) = 0\,. \tag{III.18}$$

Since the welfare function $\mathcal{W}_s$ for the supplier is identically zero in this model, the market essentially reduces to a model consisting of two players: the consumer and the network. We find that the equilibrium for the original market model provides an equilibrium for the two-agent market:

Lemma 3.6: $\{\boldsymbol{E}_D, \boldsymbol{E}_s, \boldsymbol{R}_s, \boldsymbol{P}^e\}$ is a competitive equilibrium for the $\mathfrak{S}$-market.

Proof. The triple $\{\boldsymbol{E}_D, \boldsymbol{E}_s, \boldsymbol{R}_s\}$ satisfies the supply-demand balance and the network constraints, and maximizes the consumer and the network welfare functions under price $\boldsymbol{P}^e$. Since the supplier's welfare is always zero by assumption, we can view $\{\boldsymbol{E}_s, \boldsymbol{R}_s\}$ as maximizing the supplier's welfare in the $\mathfrak{S}$-market. Thus, the lemma holds. ■

The price $\boldsymbol{P}^e$ in the original market model supports a competitive equilibrium in the $\mathfrak{S}$-market. Hence we can hope to extract properties of $\boldsymbol{P}^e$ in the simpler $\mathfrak{S}$-market model. This is the main motivation behind the introduction of the $\mathfrak{S}$-market.

Recall that in the single bus model, the derivation of the supporting price (III.14) was based on the assumption that consumers are not subject to temporal constraints. Lemma 3.7 justifies the same approach to analysis in the network model.

Lemma 3.7: All players, as well as the social planner, are myopic in the $\mathfrak{S}$-market.

Hence the optimization problems posed by the consumer and the network in the $\mathfrak{S}$-market are reduced to a "snapshot model" in which we can fix a time t to obtain properties of $P^e(t)$, exactly similar to the derivation of (III.14).

With fixed time t, the snapshot version of the SPP is given as the maximization of the total welfare $\mathcal{W}_{tot}^{\mathfrak{S}}$ with

$$\mathcal{W}_{tot}^{\mathfrak{S}} = \sum_n \Big[v_n(E_{Dn}) - c_n^{bo}(D_n - E_{Dn}) - P_n^e E_{sn}\Big]\,, \tag{III.19}$$

subject to the following constraints:

$$\begin{array}{rcll} 1^T E_S &=& 1^T E_D & \leftrightarrow \lambda\,, \\ -f_l^{\max} &\le& (E_S - E_D)^T H_l \le f_l^{\max} & \leftrightarrow \mu_l^-, \mu_l^+ \ge 0\,, \\ 0 \le E_{Dn} &\le& D_n & \leftrightarrow \eta_n^-, \eta_n^+ \ge 0\,. \end{array}$$

The terms on the right hand side are Lagrange multipliers corresponding to the given constraints. The Lagrangian of the SPP for the $\mathcal{S}$-market is the function of static variables:

$$\begin{aligned} \mathcal{L}^h = \; & -\mathcal{W}^{\mathcal{S}}_{tot} + \lambda(1^T(E_D - E_S)) \\ & + \textstyle\sum_l \mu_l^+[(E_S - E_D)^T H_l - f_l^{\max}] \\ & + \textstyle\sum_l \mu_l^-[-(E_S - E_D)^T H_l - f_l^{\max}] \\ & + \textstyle\sum_n \eta_n^+(E_{Dn} - D_n) - \sum_n \eta_n^- E_{Dn}\,, \end{aligned} \quad \text{(III.20)}$$

where $\mu_l^-, \mu_l^+, \eta_n^-, \eta_n^+ \ge 0$.

Proposition 3.8: Consider the SPP for the $\mathcal{S}$-market with welfare function defined in (III.19). Suppose that μ_l^-, μ_l^+, η_n^-, η_n^+ are the non-negative, optimal solutions to the dual with Lagrangian (III.20). Then the equilibrium price has entries given as follows: For $n = 1, \dots, N$,

$$P_n^e = \nabla v_n(E_{Dn}) + \nabla c_n^{bo}(D_n - E_{Dn}) + \Lambda\,, \quad \text{(III.21)}$$

where $P_n^e = P_n^e(t)$, E_{Dn}, D_n, E_{Dn} are also the variables observed at time t, and where

$$\Lambda = \begin{cases} 0\,, & 0 < E_{Dn} < D_n \\ -\eta_n^+\,, & E_{Dn} = D_n \\ \eta_n^-\,, & E_{Dn} = 0 \end{cases}.$$

Proof. Since $\{\boldsymbol{E}_D, \boldsymbol{E}_S, \boldsymbol{R}_S, \boldsymbol{P}^e\}$ is a competitive equilibrium for the $\mathcal{S}$-market, $\{\boldsymbol{E}_D, \boldsymbol{E}_S\}$ maximizes the SPP for the $\mathcal{S}$-market. By the KKT conditions we obtain,

$$\begin{aligned} 0 = \frac{\partial \mathcal{L}^h}{\partial E_{Dn}} &= -\nabla v_n(E_{Dn}) - \nabla c_n^{bo}(D_n - E_{Dn}) \\ & \quad + \lambda + \sum_l (\mu_{ln}^- - \mu_{ln}^+) \cdot H_{ln} + \eta_n^+ - \eta_n^-\,. \end{aligned}$$

$$0 = \frac{\partial \mathcal{L}^h}{\partial E_{sn}} = P_n^e - \lambda - \sum_l (\mu_{ln}^- - \mu_{ln}^+) \cdot H_{ln}\,.$$

On summing these two equations we obtain,

$$P_n^e = \nabla v_n(E_{Dn}) + \nabla c_n^{bo}(D_n - E_{Dn}) - \eta_n^+ + \eta_n^-\,.$$

The proposition then follows from the complementary slackness conditions. ■

Note (III.21) holds for all equilibria in the $\mathcal{S}$-market; consequently, it holds for all equilibria in the original market. It appears in (III.21) that prices depend upon the actions of the players. This is not the case. Similar to the Proposition 3.1, we can write the price as $P_n^e(t) = \nabla v_n(E_{Dn}^*(t)) + \nabla c_n^{bo}(D_n(t) - E_{Dn}^*(t)) + \Lambda^*(t)$, where $\{E_{Dn}^*(t)\}$ (and consequently $\Lambda^*(t)$) are obtained as the solution of the SPP.

To apply the proposition we must identify the parameters $\{\eta_n^-, \eta_n^+\}$. This is possible since they are precisely the sensitivities of the SPP with respect to the constraints $E_{Dn} \ge 0$ and $E_{Dn} \le D_n$ respectively. We motivate the price computations through numerical examples in next section.

IV. Numerical Illustrations

We now present examples to illustrate the application of Proposition 3.8. In the example shown in Figure 2 we explain why prices can be negative, or go well above the choke-up price. We then consider prices in a market setting where two types of services – *primary* and *ancillary* – are provided. The model resembles the power network model of [7], but we ignore transmission constraints, and make suitable extensions to capture demand response. We find that the variance of prices can be reduced dramatically with demand response.

A. Texas model

Consider the 3-bus network shown in Figure 2, with injection shift factor matrix H given by the expression (II.8). We assume that supplier is located at bus 1 while buses 2 and 3 are load buses. Each of the buyers at buses 2 and 3 has linear utility of consumption and disutility of blackout (II.1b), with common parameters v, c^{bo}.

We use the $\mathcal{S}$-market in Section III-B for analysis. Recall that the supplier's welfare function is identically zero in this setting. Suppose that $D_2 = 170$ MW and $D_3 = 30$ MW, and the utility and disutility functions are both linear. Then, the snapshot SPP for the $\mathcal{S}$-market is,

$$\begin{array}{ll} \min & -[v(E_{D2} + E_{D3}) - c^{bo}(200 - E_{D2} - E_{D3})] \\ \text{s.t.} & E_{S1} = E_{D2} + E_{D3} \\ & -f_{12}^{\max} \le \frac{2}{3}E_{D2} + \frac{1}{3}E_{D3} \le f_{12}^{\max} \\ & -f_{23}^{\max} \le \frac{1}{3}E_{D2} - \frac{1}{3}E_{D3} \le f_{23}^{\max} \\ & -f_{13}^{\max} \le \frac{1}{3}E_{D2} + \frac{2}{3}E_{D3} \le f_{13}^{\max} \\ & 0 \le E_{D2} \le 170\,,\ 0 \le E_{D3} \le 30\,. \end{array}$$

We assume that the supporting price P_1^e at bus 1 is *zero*, and that this is true not just for the snapshot values $\{E_{Di},\ E_{Si},\ R_{Si}\}$ and parameters $\{f_{ij}^{\max}\}$, but for all values in a neighborhood of these nominal values. This is not unreasonable given the results of Section III-A, provided that reserves are strictly positive at bus 1. Under these assumptions we can then compute the prices P_2^e and P_3^e at buses 2 and 3 respectively.

Negative prices: Assume that $f_{23}^{\max} = 40$ MW, while the other two lines are unconstrained. Solving the SPP for the $\mathcal{S}$-market we obtain $E_{D2} = 150$ MW and $E_{D3} = 30$ MW, and we find that the constraint $f_{23}^{\max} = 40$ MW is reached. Since $0 < E_{D2} < D_2$, we have $P_2^e = v + c^{bo}$.

For a given $\epsilon \in \mathbb{R}$, we perturb the constraint on E_{D3} to obtain $0 \le E_{D3} \le 30 + \epsilon$. On re-solving the SPP we obtain $E_{D2} = 150 + \epsilon$ MW and $E_{D3} = 30 + \epsilon$ MW. Applying Proposition 3.8, we conclude that P_3^e is given by the limit,

$$v + c^{bo} + \lim_{\epsilon \to 0} \frac{-(180 + 2\epsilon)v + (20 - 2\epsilon)c^{bo} + 180v - 20c^{bo}}{\epsilon}\,.$$

That is, $P_3^e = -(v + c^{bo})$, which is clearly *negative*.

Prices exceeding the choke up price: Assume that $f_{13}^{\max} = 50$ MW, while the other two lines are unconstrained. Again, solving the SPP for the $\mathcal{S}$-market gives $E_{D2} = 150$ MW and $E_{D3} = 0$ MW with the constraint $f_{13}^{\max} = 50$ MW being reached. Proposition 3.8 gives $P^e_2 = v + c^{\text{bo}}$ since $0 < E_{\text{D}2} < D_2$.

For a given $\epsilon \in \mathbb{R}$, we perturb the constraint on $E_{\text{D}3}$ to obtain $0 + \epsilon \leq E_{\text{D}3} \leq 30$. On re-solving the SPP, we obtain $E_{\text{D}2} = 150 - 2\epsilon$ MW and $E_{\text{D}3} = \epsilon$ MW. We conclude that P^e_3 is again expressed as a limit,

$$v + c^{\text{bo}} + \lim_{\epsilon \to 0} \frac{-(180-\epsilon)v + (20+\epsilon)c^{\text{bo}} + 180v - 20c^{\text{bo}}}{\epsilon} \,.$$

That is, $P^e_3 = 2(v + c^{\text{bo}})$, which is twice the choke-up price.

B. Ancillary service

The results presented in the preceding sections can be adapted to a market setting in which a number services can be used to meet the demand. We consider the model of [8] in which $G^{\text{p}}(t)$ and $G^{\text{a}}(t)$ denote the instantaneous output of primary and ancillary generators at time t. Primary service takes on positive or negative values since it represents deviations from day-ahead schedules.Ancillary service is constrained to be non-negative. In addition, the two sources of generation are distinguished by their *ramping* capabilities: $\zeta^{\text{p}+}$, $\zeta^{\text{a}+}$, $\zeta^{\text{p}-}$ and $\zeta^{\text{a}-}$ represent the maximum rates for ramping up and down the primary and ancillary services, respectively. For simplicity, we focus on the case in which ramping down is unconstrained, i.e., $\zeta^{\text{p}-}, \zeta^{\text{a}-} = \infty$. We assume that $\zeta^{\text{a}+} > \zeta^{\text{p}+}$, to reflect the ability of ancillary service to ramp up faster than primary.

The demand at time t is given as $D(t)$ and the reserve at that time is given by,

$$R(t) = G^{\text{p}}(t) + G^{\text{a}}(t) - D(t), \quad t \geq 0\,. \tag{IV.22}$$

In our model, we consider some *demand-response* capabilities; that is certain loads can be turned off to maintain supply-demand balance in the event of reserves shortfall. When the demand exceeds the available supply capacity, demand response capacity is deployed first to balance the demand and supply. If the difference between the demand and available supply is less than the available demand response capacity – i.e. demand response capacity can sufficiently cover the reserve shortfall – the price is set by the cost of demand response. Otherwise, forced blackout will take place, in which case the price equals the choke-up price. Since the cost of demand response is typically lower than the cost of blackout, the price when the supply deficiency can be covered by demand response would be lower than that if demand response was unavailable. Thus, demand response acts as a cushion between the normal security operations and the blackout. In our model, we use $\bar{r}^{\text{DR}}_{\max}$ to denote total demand response capacity. That is, $\bar{r}^{\text{DR}}_{\max}$ is the threshold up to which load can be shed. Hence, we have black-out situations only if $R(t) \leq -\bar{r}^{\text{DR}}_{\max}$.

In this example the SPP can be converted to a cost minimization problem, in which the cost function on $(\boldsymbol{G}^{\text{p}}, \boldsymbol{G}^{\text{a}}, \boldsymbol{R})$ has the following form: For $t \geq 0$, $\mathcal{C}(t) :=$

$$c^{\text{p}} G^{\text{p}}(t) + c^{\text{a}} G^{\text{a}}(t) + (c^{\text{bo}} - c^{\text{dr}}) \mathbb{I}(R(t) < -\bar{r}^{\text{DR}}_{\max}) + c^{\text{dr}} \mathbb{I}(R(t) < 0)$$

where c^{p}, c^{a} represent the per unit production costs of primary and ancillary services respectively, c^{dr} is the cost of demand response-based load shedding and c^{bo} is the cost of blackout [8].

Implications of these results for ancillary service providers We illustrate the impacts using simulations based on a controlled random-walk (CRW) model. That is, the demand is modeled as a random walk of the form,

$$D(k+1) = D(k) + \mathcal{E}(k+1), \qquad k \geq 0, \quad D(0) = 0\,,$$

in which the increment process $\boldsymbol{\mathcal{E}}$ is a bounded i.i.d. sequence. The reserve $\boldsymbol{R}$ is modeled in discrete time using the expression (IV.22).

The other model parameters used are as follows: $c^{\text{p}} = 1$, $c^{\text{a}} = 20$, $c^{\text{dr}} = 100$ and $c^{\text{bo}} = 400$. The ramp-up rates are taken as $\zeta^{\text{p}+} = 1/10$ and $\zeta^{\text{a}+} = 2/5$. The marginal distribution of the increment distribution was taken symmetric on $\{\pm 1\}$.

We perform experiments based on a family of threshold policies. Such policy is based on two thresholds $(\bar{r}^{\text{p}}, \bar{r}^{\text{a}})$. Under the policy considered, primary service is ramped up whenever $R(t) \leq \bar{r}^{\text{p}}$. Similarly, ancillary service is ramped up whenever $R(t) \leq \bar{r}^{\text{a}}$. We refer the reader to [7], [16] further details about the threshold policy.

In the numerics we considered discounting as in (II.11), but in discrete time, with discount factor $\beta = 0.995$. We find the "best-threshold" by estimating the discounted cost by the standard Monte-Carlo estimate. We perform several experiments with varying levels of demand response. We use c^{dr} and $\bar{r}^{\text{DR}}_{\max}$ as simulation parameters to study sensitivity of average prices to demand response capabilities.

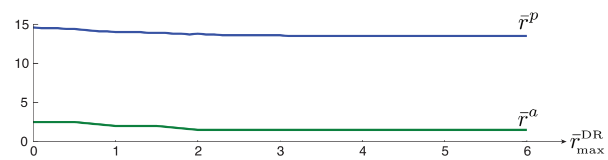

Fig. 3. Optimal thresholds for primary and ancillary service

Figure 3 plots the optimal thresholds for primary and ancillary service with respect to the demand response capacity $\bar{r}^{\text{DR}}_{\max}$. The optimal threshold for primary service is much higher than that of ancillary service, which reflects the fact that the primary service ramps up slower than ancillary service, and primary service is less expensive. The low sensitivity is consistent with the conclusions of [8].

The average prices for primary service, and the conditional average of ancillary service are shown in Figure 4, for various values of $\bar{r}^{\text{DR}}_{\max}$. The average price $\mathsf{E}[P^e] = c_{\text{p}}$ is consistent with the conclusions of [6] or Theorem 3.4 (recall that G^{p} is not sign-constrained). The conditional mean "$\mathsf{E}[P^e \mid G^{\text{a}} > 0]$" denotes

$$\Big[\int_0^\infty e^{-\gamma t} P^e(t) \mathbb{I}(G^{\text{a}}(t) > 0)\, dt\Big] \Big[\int_0^\infty e^{-\gamma t} \mathbb{I}(G^{\text{a}}(t) > 0)\, dt\Big]^{-1}.$$

The results shown are consistent with (III.16): It appears that $\nu^* \approx 5\Big(\int_0^\infty e^{-\gamma t} \mathbb{I}(G^{\text{a}}(t) > 0)\, dt\Big)$ when $\bar{r}^{\text{DR}}_{\max} \geq 3$.

Also shown in Figure 4 is a plot of the variance of the equilibrium price for with respect to $\bar{r}^{\text{DR}}_{\max}$. We see that the variance drops dramatically with an increase in the demand

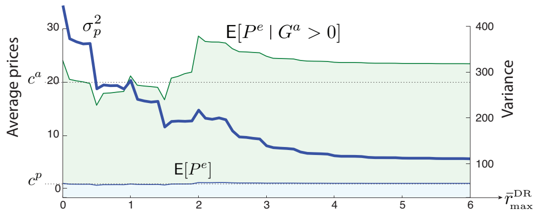

Fig. 4. Average prices and variance of P^e.

response capacity, even though the optimal thresholds are virtually unchanged.

Recall that the prices for primary and ancillary service are identical. However, the bulk of primary service is allocated in the day ahead market. Consequently, ancillary service, such as provided by gas turbines, will be exposed to much greater variability in the efficient market outcome.

V. Conclusions

Once dynamics and uncertainty are brought into play, the competitive equilibrium of RTMs is characterized by volatile and high prices. In the competitive equilibrium, the variance of income to suppliers is high, while the average price coincides with the average marginal cost. This creates significant barriers to entry for peaking units that derive little benefit from DAM participation.

The competitive equilibrium is an extreme idealization of the real-world. It gives insight, such as an explanation for difficulties in providing incentives for peaking units in today's markets. However, the competitive equilibrium is a crude predictive model, and certainly far too crude for long-term predictions of system behavior.

The power grid is as important to our society as our transportation systems or healthcare systems. Grid reliability is critical — blackouts can have tremendous social and economic costs, wasting all the "efficiency" improvements and gains of several years. The best architecture for the energy highways of the future is not yet obvious to us, but it is likely to include elements of todays DAM combined with long-term contracts. However, it is obvious that we will need lanes and speed limits, incentives and penalties, in order to achieve a reliable system.

We strongly believe that a new paradigm for the design and operation of future energy markets is required. It is possible that in a few years all of the smart meters and wind farms installed today will be regarded as another "bridge to nowhere" unless we create the right architecture to make use of these resources, which must include reliable market mechanisms. In particular, we must move beyond traditional static competitive equilibrium analysis, and recognize the impact of dynamics and volatility. To this end, many of the issues surveyed here require the application of successful power and energy methodologies of the past, complemented with approaches from other disciplines such as decision and control theory, simulation and learning. While, efficiency remains a key metric in design, we need to bring further objectives into the fold such as sustainability and reliability. Finally, the possibly adverse role of strategic interactions cannot be overstated and presents yet another challenge.

We hope the vision and results presented in this paper can contribute to the ongoing process of bridging the decision & control, and power & energy systems communities, reflected by several recent works on this boundary [6]–[11], [16], [18]–[20].

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