The Value of Volatile Resources in Electricity Markets

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Abstract— **While renewable resources most certainly provide environmental benefits, and also help to meet aggressive renewable energy targets, their deployment has pronounced impacts on system operations. There is an acute need to understand these impacts in order to fully harness the benefits of renewable resource integration. In this paper we focus on the integration of wind energy resources in a multi-settlement electricity market structure. We study the dynamic competitive equilibrium for a stochastic market model, and obtain closed form expressions for the supplier and consumer surpluses. Numerical results based on these formulae show that the value of wind generation to consumers, under the current operational practices, falls *dramatically* with volatility. In fact, we can establish thresholds for the coefficient of variation beyond which the value of wind is questionable. These findings emphasize the need to investigate operational schemes that address volatility.**

I. INTRODUCTION

Growing environmental concerns and possible introduction of carbon regulation have led many states in the United States to adopt renewable portfolio standards and other similar policies which mandate that a certain percentage of electricity production must come from renewable resources [1], [2]. Among the many renewable energy sources, wind power is the most attractive for massive deployment due to reduced investment costs, rapid installation, and low operational/maintenance costs [3]. These factors coupled with the legislative stimulus available for wind installation projects have resulted in an exponential growth in installed wind generation capacity over the last decade [4]. However, its deployment presents major challenges in system operations due to the inherent characteristics of wind generation: limited control capabilities, forecasting uncertainty, and intermittency in the generation outputs [5]. To overcome these challenges and fully harness the benefits associated with wind resource integration and deployment, we need to understand the unique characteristics of wind generation and the overall impact on the power system and market operations.

The operation of a power system and its electricity markets is a complex task because of the variability in electricity supply and demand, the physical constraints on the system resources, the large-scale nature of the system, and the various sources of uncertainty. Such complexities are further compounded in a market environment driven by private interests as well as regulatory policies [6]. These factors coupled with the interconnectedness of the power system require a centralized coordination process to dispatch the supply resources and meet the demand requirements. In most jurisdictions, an independent entity known as the independent system operator (ISO) is in charge of this centralized coordination process. The ISO is responsible for operating the system and its electricity markets in a reliable and economic manner.

The ISO typically maintains extra generation capacity online at all times as reserves to ensure that electricity supply is reliable in-spite of uncertainty as well as variability in both demand and supply. Fig. 1 illustrates the reserve policy of ISO New England: Scheduled generation is roughly 4GWs greater than the day-ahead forecast throughout the day.

Fig. 1: August 30, 2010 ISO-NE day-ahead and real-time demand and supply (www.iso-ne.com).

With a low penetration of wind resources, the demand and supply exhibit fairly well-understood patterns, so that the evolution of system and market conditions are predictable. As the deployment of wind resources increases, the resulting increase in uncertainty will force the ISO's to adjust their operating policies to ensure reliability of electricity supply.
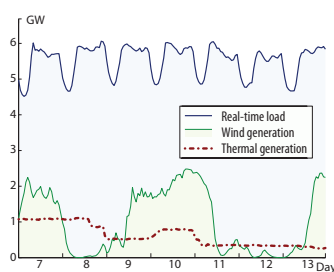

Fig. 2: Wind generation data from BPA balancing authority (www.transmission.bpa.gov).

Shown in Fig. 2 is the total load as well as the available wind and thermal generation in the balancing area of Bonneville Power Administration for the week of June 7 - 13, 2010. The plot emphasizes the diurnal nature of the demand. We also note that the variability of available thermal generation is low; the sudden jumps in thermal generation arise from start-up or shut-down of units as a result of the day-ahead commitment decisions. However, the variability in the minute-by-minute wind generation as well as the daily averages are much more pronounced. Clearly, deep penetration of wind resources will exacerbate variability as well as uncertainty of the supply resources. Reliability requirements will then necessitate the procurement of additional reserves. These impacts of wind generation are well recognized, and attempts to quantify them have been pursued using time-series based modeling techniques [7]–[9]. It is strongly implied in [10] and [11] that the introduction of wind resources will require new ways of thinking about market operations. It is argued in [11] that the allocation

of risk and uncertainty associated with wind generation, regardless of whether it is used as a supply-side resource or a demand-side resource, is a challenging problem. Analysis of dynamic stochastic models to address these questions is presented in [12]–[15] (and earlier work of Caramanis et. al. referenced in [14]).

In this paper we evaluate the impact of wind generation by focusing on two parameters: penetration and volatility. Our goal is to understand how the low cost of wind generation can be offset by the impacts associated with its volatility. In order to accomplish this, we extend the dynamic modeling approach of [12], [13] to represent the inter-related day-ahead and real-time electricity markets. In [13], the analogy with manufacturing systems is exploited to quantify optimal reserves in an electricity market. An exact expression for the optimal reserves is obtained, whose value grows linearly with the variance of demand.

We argue in this paper that volatility in supply has identical consequences if the currently adopted 'use all the wind' policy is utilized. Based on this argument, we can expect to see increased reserves and pronounced impacts on market outcomes as the wind penetration deepens. These conclusions are quantified by extending the stochastic market model of [13] to explicitly represent the interplay between day-ahead commitment decisions and real-time operations. We study the dynamic competitive equilibrium for such coupled markets and obtain closed form expressions for supplier and consumer surpluses. We compute the expected surpluses under different wind commanding schemes: wind resources can be commanded by either consumers or suppliers. The numerical experiments illustrate the critical role of volatility in determing the value of wind generation to consumers under current operational paradigms. Our model can be used to establish thresholds for the coefficient of variation beyond which the value of wind is questionable.

The remainder of the paper contains three additional sections. In § II we describe the multi-settlement electricity market model, and survey some of its features. We devote § III to a refinement of this model that incorporates wind generation. We illustrate numerically the impacts of wind volatility on the market outcomes, and provide insights on various wind resource integration schemes. We provide concluding remarks and suggestions for future research in § IV.

II. ELECTRICITY MARKETS MODELS

In many jurisdictions around the world, electricity is traded using two or more interrelated markets that constitute the so-called *multi-settlement structure*. Although each such market trades in the *MWh* commodity, they are differentiated by the time intervals between the trading decisions and the energy delivery, which range from year(s), month(s), day, hour(s), to minutes. There are economic and operational reasons for adopting such a multi-settlement structure. Forward contract markets, which trade energy years and months in advance, create signals for resource investments and ensure adequacy of resources. The day-ahead market (DAM) allows sellers

and buyers to hedge against the risk associated with the uncertain real-time conditions, which is argued to improve market efficiency [16]. Also, from an operational point of view, the physical constraints on the generation units – ramping limitations and start-up constraints – make it impossible to support the "just-in-time" electricity production within a market structure which uses only real-time trading. The clearing of the DAM one day prior to the actual production and delivery of energy allows the ISO to schedule generation in such a way that physical constraints are met. As supply and demand are not perfectly predictable in the DAM, the real-time market (RTM) – which is operated minutes-ahead of the actual "real time" – allows fine-tuning of the resource allocation decisions made in the DAM. The RTM is typically cleared every 5 to 10 minutes, so as to maintain a continuous balance between supply and demand.

The market model introduced in this paper is based on the multi-settlement structure, consisting of DAMs and RTMs, that is most commonly adopted by ISOs today.

In what follows we present the main design elements used in our analysis to represent the coupling of the DAM and the RTM. While real-world DAM and RTM clearing operates at discrete decision epochs, we construct continuous time models to approximate the coupled markets. The models are constructed in a stochastic setting to capture uncertainty in supply and demand.

A. Day-ahead market model

Our model captures the result of DAM clearing on the generation scheduling. The reliability concerns are considered through the reserve policy, which we assume is a part of the DAM clearing. That is, each supplier offers a bundle of energy and capacity which contributes towards meeting the forecasted demand and reserve requirements. Also, to simplify the analysis, we assume that the transmission grid has ample capacity to support the commitment decisions.

We assume that the market clearing occurs continuously. Time $t = 0$ signals the start of the 'new day' in the DAM. We use $d^{\mathrm{da}}(t)$ to denote the forecasted demand at time t, and $p^{\mathrm{da}}(t)$ to denote the market clearing price for the DAM. The generation schedule is determined from the DAM which is cleared to meet the demand and reserve requirements in an economic manner. We use $g^{\mathrm{da}}(t)$ to represent the promised supply schedule obtained in the DAM. The forecast reserves are thus,

$$r^{\mathrm{da}}(t) = g^{\mathrm{da}}(t) - d^{\mathrm{da}}(t), \quad t \geq 0 \quad . \tag{1}$$

The fixed reserve policy typically adopted by the ISOs justifies our standing assumption,

$$r^{\mathrm{da}}(t) \equiv r^{\mathrm{da}}_0 \quad \text{is } constant. \tag{2}$$

B. Real-time market model

Here we adapt the dynamic market model of [13] to represent the continuous time RTM model. The reserves form a state-process for the optimization problems considered by the consumer and supplier. As in the DAM, $t = 0$ represents

the start of the 'new day' in the RTM. For each $t \geq 0$, the total demand is denoted $D^{\mathrm{ttl}}(t) = d^{\mathrm{da}}(t) + D(t)$ and the total capacity is $G^{\mathrm{ttl}}(t) = g^{\mathrm{da}}(t) + G(t)$, and in view of (2) the total reserve at time t is thus,

$$R^{\mathrm{ttl}}(t) = G^{\mathrm{ttl}}(t) - D^{\mathrm{ttl}}(t) = G(t) - D(t) + r_0^{\mathrm{da}}. \quad (3)$$

It is likely that the generation $G(t)$ is obtained from different sources than $g^{\mathrm{da}}(t)$. For example, gas-turbine generators may be expensive to operate, but they can be ramped up more rapidly than coal or nuclear generation.

Observe that the initial condition $R^{\mathrm{ttl}}(0) = G(0) + r_0^{\mathrm{da}} - D(0)$ contains two components from the two markets, and the uncertain initial deviation in forecast demand. If $G(0) > 0$, this might mean that gas turbines are spinning, and ready to deliver power.

The stochastic model considered in [12], [13] consists of the following components:

(i) *Volatility* The analysis of [13] is restricted to the case where the deviation in demand $\boldsymbol{D}$ is modeled as Brownian motion: $\boldsymbol{D}$ is a driftless Brownian motion with instantaneous variance σ^2.

(ii) *Friction* Since generation cannot increase instantaneously, there exists $\zeta \in (0, \infty)$ such that,

$$\frac{G(t') - G(t)}{t' - t} \leq \zeta, \quad \text{for all } t \geq 0, \text{ and } t' > t. \quad (4)$$

No corresponding lower bound is imposed.
Under these assumptions, we may view the reserve process as a controlled stochastic system, which we write as the SDE,

$$dR(t) = \zeta dt - dI(t) - dD(t) \quad (5)$$

where $\boldsymbol{I}$ is non-decreasing. That is, we are writing $dG(t) = \zeta dt - dI(t)$ to model the upper bound on the rate of increase in generation.

The market analysis of the model (5) in [13] is based on the following assumptions:

(i) *Cost* The production technology of the supplier is subject to a production cost $c(G(t))$ for the production capacity $G(t)$ made available at time $t \geq 0$. The cost incurs, regardless of the delivery of power. The cost is a linear function of $G(t)$, of the form $cG(t)$ for some constant $c > 0$.

(ii) *Value of power* For each unit of power delivered, the consumer obtains v units of utility. Thus, the utility of the consumer is $v \min(D(t), G(t))$.

(iii) *Disutility from power loss* If the demand is not met ($R(t) < 0$), the consumer suffers utility loss $c^{\mathrm{bo}} R(t)$ for some $c^{\mathrm{bo}} > 0$.

(iv) *Perfect competition* The price of power $P(t)$ in the RTM is assumed to be exogenous – it is independent of the decisions of the market players.

The objectives of the consumer and the supplier are specified by the respective welfare functions,

$$\mathcal{W}_{\mathrm{s}}(t) := P(t)G(t) - cG(t)$$

$$\mathcal{W}_{\mathrm{D}}(t) := v \min(D(t), G(t)) \quad (6)$$
$$- c^{\mathrm{bo}} \max(0, -R(t)) - P(t)G(t)$$

The consumer and supplier each optimize the discounted mean-welfare. For given initial values of generation $G(0) = g$ and demand $D(0) = d$, the respective discounted rewards are denoted $K_{\mathrm{s}}(g, d) := \mathsf{E}\left[\int e^{-\gamma t} \mathcal{W}_{\mathrm{s}}(t) \, dt\right]$ and $K_{\mathrm{D}}(g, d) := \mathsf{E}\left[\int e^{-\gamma t} \mathcal{W}_{\mathrm{D}}(t) \, dt\right]$, where $\gamma > 0$ is the discount rate.

A closed form solution for the unique market equilibrium is obtained in [13]. The price of power is expressed as a static function of the equilibrium reserve $R^e(t)$: the *equilibrium price functional* is a piecewise constant function of the equilibrium reserve process,

$$p^e(r^e) = (v + c^{\mathrm{bo}})\mathbb{I}\{r^e < 0\} \quad (7)$$

The sum $v + c^{\mathrm{bo}}$ is in fact the maximum price the consumer is willing to pay, often called the *choke-up price*. Subject to the price $P^e(t) = p^e(R^e(t))$, it is clear why the consumer will wish to keep reserves in order to avoid paying the choke-up price.

The social planner's problem is defined as the minimization of the discounted mean of the sum,

$$K(g, d) = \mathsf{E}\left[\int e^{-\gamma t} \big(\mathcal{W}_{\mathrm{s}}(t) + \mathcal{W}_{\mathrm{D}}(t)\big) \, dt\right].$$

The optimal solution is obtained in [13], following [17], defined so that the resulting reserve process is a reflected Brownian motion (RBM) on the half-line $(-\infty, \bar{r}^*]$, with

$$\bar{r}^* = \frac{1}{\theta_+} \log\left(\frac{v + c^{\mathrm{bo}}}{c}\right). \quad (8)$$

where θ_+ is the positive solution to the quadratic equation $\frac{1}{2}\sigma^2\theta^2 - \zeta\theta - \gamma = 0$. For $\gamma \sim 0$ we have $\theta_+ \approx 2\zeta/\sigma^2 + \gamma/\zeta$. It is also shown in [13] that the solution to the average cost case case is obtained as the limit of the discount-cost solution, as $\gamma \downarrow 0$.

From (8), we see that in a system with volatile demand and ramping-constrained supply, the optimal reserve threshold is directly proportional to variance σ^2 and inversely proportional to ramping rate ζ. While this result seems intuitively correct, it is clear from Fig. 2 that we need to consider volatility from both supply and demand if we are to include resources such as wind. We discuss the corresponding extensions to the RTM model to capture these features in § III.

C. Multi-settlement market model

We now describe a coupling of the DAM and RTMs that defines the multi-settlement market model. We maintain our assumption (2) that reserves in the DAM are constant, $r^{\mathrm{da}}(t) \equiv r_0^{\mathrm{da}}$. Also, r_0^{da} and $G(0)$ are constrained to be non-negative. We assume that a prior distribution μ_D on $\mathbb{R}$ is given with $D(0) \sim \mu_D$, and that $D(0)$ is independent of future increments in demand, so that $\boldsymbol{D}$ remains a stochastic process with independent-increments. Due to lack of space, throughout most of the paper we specialize to the ideal case in which $D(0) \equiv 0$. This approximation is reasonable if $t = 0$ corresponds to midnight, at which time demand is highly predictable.

We assume that the equilibrium solution is obtained in the RTM: we denote by $K_{\mathrm{D}}^*(g, d)$, $K_{\mathrm{s}}^*(g, d)$ the discounted

mean-welfare functions in the RTM equilibrium outcome with $G(0) = g$ and $D(0) = d$. With $D(0) \equiv 0$, the optimal discounted mean-welfare functions are only a function of $r = R(0) = G(0) + r_0^{\text{da}}$. We then simplify notation, writing

$$
\begin{aligned}
K_{\text{s}}^*(r) &:= \mathsf{E}\Big[\int e^{-\gamma t}\mathcal{W}_{\text{s}}^*(t)\,dt\Big], \\
K_{\text{D}}^*(r) &:= \mathsf{E}\Big[\int e^{-\gamma t}\mathcal{W}_{\text{D}}^*(t)\,dt\Big].
\end{aligned}
\tag{9}
$$

1) Consumer welfare: Consider first the complete optimization problem posed by the consumer, based on welfare

$$
\begin{aligned}
\mathcal{W}_{\text{D}}^{\text{ttl}}(t) := v\min(D^{\text{ttl}}(t), G^{\text{ttl}}(t)) - c^{\text{bo}}\max(0, -R(t)) \\
- P^e(t)G(t) - p^{\text{da}}(t)g^{\text{da}}(t)
\end{aligned}
$$

where $P^e(t)$ is the market clearing price. Recalling the definition of $\mathcal{W}_{\text{D}}(t)$ in (6), routine calculations give

$$
\mathcal{W}_{\text{D}}^{\text{ttl}}(t) = \mathcal{W}_{\text{D}}(t) + \Big\{(v - p^{\text{da}}(t))d^{\text{da}}(t) + (P^e(t) - p^{\text{da}}(t))r_0^{\text{da}}\Big\}
$$

Under our assumption that $D(0)$ is independent of future increments in demand, the total discounted mean welfare is given by

$$
\begin{aligned}
K_{\text{D}}^{\text{ttl}} = \int K_{\text{D}}^*(g, z)\,\mu_D(dz) \\
+ \int_0^\infty \mathsf{E}\Big[(v - p^{\text{da}}(t))d^{\text{da}}(t) + (P^e(t) - p^{\text{da}}(t))r_0^{\text{da}}\Big]e^{-\gamma t}\,dt
\end{aligned}
\tag{10}
$$

The convention that the reserves in the DAM (1) are constant will help to simplify this expression. In addition, for purposes of computation, we henceforth take $\mu_D = \delta_0$ so that $D(0) \equiv 0$. The expectation in (10) is computed in the Appendix using

$$
\overline{p}^e(r) := \gamma\int_0^\infty \mathsf{E}\big[P^e(t) \mid R(0) = r\,, D(0) = 0\big]e^{-\gamma t}\,dt
$$

In [13], this is shown to be a non-increasing function of r, satisfying $c^{\text{rt}} \le \overline{p}^e(r) \le v + c^{\text{bo}}$ for each $r \in (-\infty, \bar{r}^*]$, and $\overline{p}^e(\bar{r}^*) = \min_r \overline{p}^e(r) = c^{\text{rt}}$. When $D(0) = 0$, the total discounted mean welfare is thus simplified to $K_{\text{D}}^{\text{ttl}} = K_{\text{D}}^*(r) + \int_0^\infty\big\{(v - p^{\text{da}}(t))d^{\text{da}}(t) + (\overline{p}^e(r) - p^{\text{da}}(t))r_0^{\text{da}}\big\}e^{-\gamma t}\,dt$.

Let $\overline{p}^{\text{da}}$ denote the average price in the DAM; $\overline{p}^{\text{da}} = \gamma\int_0^\infty p^{\text{da}}(t)\,e^{-\gamma t}\,dt$. On substituting the expression $r_0^{\text{da}} = R(0) - G(0) = r - G(0)$, we obtain

$$
\begin{aligned}
K_{\text{D}}^{\text{ttl}} = K_{\text{D}}^*(r) + \gamma^{-1}(r - G(0))(\overline{p}^e(r) - \overline{p}^{\text{da}}) \\
+ \int_0^\infty (v - p^{\text{da}}(t))d^{\text{da}}(t)\,e^{-\gamma t}\,dt\,.
\end{aligned}
\tag{11}
$$

2) Supplier welfare: The total supplier welfare is obtained through similar and simpler arguments. We have $\mathcal{W}_{\text{s}}^{\text{ttl}}(t) = \mathcal{W}_{\text{s}}(t) + \big(p^{\text{da}}(t) - c^{\text{da}}\big)g^{\text{da}}(t)$, with $\mathcal{W}_{\text{s}}(t)$ defined in (6). On substituting $g^{\text{da}}(t) = d^{\text{da}}(t) + r - G(0)$ and integrating, we get

$$
\begin{aligned}
K_{\text{s}}^{\text{ttl}} = K_{\text{s}}^*(r) + \gamma^{-1}(r - G(0))(\overline{p}^{\text{da}} - c^{\text{da}}) \\
+ \int_0^\infty \big(p^{\text{da}}(t) - c^{\text{da}}\big)d^{\text{da}}(t)\,e^{-\gamma t}\,dt\,.
\end{aligned}
\tag{12}
$$

Closed form expressions for $K_{\text{D}}^*(g, d)$ and $K_{\text{D}}^*(g, d)$ can be obtained for arbitrary (g, d). Computation is simplified when $D(0) = 0$ since the mean of $D(t)$ is zero for each t. On substituting the price into the respective welfare functions defined in (6) we obtain in this case,

$$
\begin{aligned}
\mathsf{E}[\mathcal{W}_{\text{s}}^*(t)] = (v + c^{\text{bo}})\big(\mathsf{E}[D(t)\mathbb{I}\{R^e(t) \le 0\}] \\
+ \mathsf{E}[R^e(t)\mathbb{I}\{R^e(t) \le 0\}]\big) - c\mathsf{E}[R^e(t)]
\end{aligned}
\tag{13}
$$

$$
\mathsf{E}[\mathcal{W}_{\text{D}}^*(t)] = -(v + c^{\text{bo}})\mathsf{E}[D(t)\mathbb{I}\{R^e(t) \le 0\}]
\tag{14}
$$

where the two welfare functions are defined with the price functional p^e given in (7) and where $\boldsymbol{R} = \boldsymbol{R}^e$ is the equilibrium reserve process. Computation of the two value functions in (9) is performed in the Appendix based on these representations.
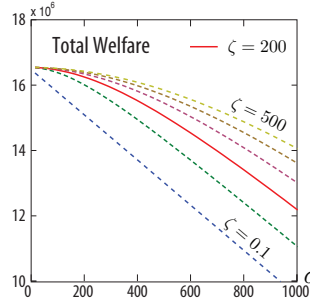

Fig. 3: Social planner's welfare.

Fig. 3 shows plots of the *total welfare*, $K_{\text{D}}^{\text{ttl}} + K_{\text{s}}^{\text{ttl}}$, for $d = 0$, and $g = \bar{r}^*$, for $\zeta = 0.1, 100, 200, 300, 500$, and various values of σ. The values for the other parameters are provided in § III-C (see eq. (15)). As expected, the welfare decays rapidly with decreased ramping rate ζ or increased volatility.

III. WHO COMMANDS THE WIND?

We now extend the DAM/RTM model to differentiate between the generation of wind resources and that of conventional resources. A key assumption used in our analysis is that all the wind generation available is dispatched and injected into the system. It follows that conventional generators serve the residual demand. The volatility of the residual demand is likely to increase due to the volatility of wind generation when compared with that of the original demand and this will result in procurement of higher reserves in the dynamic market equilibrium. The impacts depend on both volatility of wind resources and their proportion in the overall generation. With low penetration of wind resources, the increase in demand volatility will be negligible, and hence the impact on the market outcome will not be significant. Potential negative market outcomes are possible with a combination of high wind resource penetration and high volatility of wind generation.

To quantify these claims, we obtain in this section expressions for the total welfare, differentiated by who commands the wind resources: the consumer or the supplier. We obtain closed form expressions for the discounted mean welfare of the consumer and supplier in each of the two settings. Perhaps surprisingly, in the numerical results that follow we find that the supplier can achieve significant gains *even when the consumer commands the wind*. The explanation is that the higher volatility forces the consumer to pay for higher reserves in the DAM. We conclude this section with discussion on how wind power can be integrated into electricity

markets to ensure reliability, efficiency, and fairness to both consumers and suppliers.

A. Consumers command the wind

We first consider a setting in which the wind resources are commanded by the demand-side. The total wind capacity is denoted by $G_{\mathrm{w}}^{\mathrm{ttl}}(t) = g_{\mathrm{w}}^{\mathrm{da}}(t) + G_{\mathrm{w}}(t)$. The consumer surplus at time t is thus given by,

$$\begin{aligned}\mathcal{W}_{\mathrm{D,w}}^{\mathrm{ttl}}(t) = {}& v \min(D^{\mathrm{ttl}}(t), G^{\mathrm{ttl}}(t) + G_{\mathrm{w}}^{\mathrm{ttl}}(t)) \\ & - c^{\mathrm{bo}} \max(0, -R(t)) - P(t)G(t) - p^{\mathrm{da}}(t)g^{\mathrm{da}}(t)\end{aligned}$$

As in [11], we model the resulting market by interpreting wind generation as a negative load. We denote the resulting residual demand by $D^{\mathrm{net}}(t) = D(t) - G_{\mathrm{w}}(t)$ and obtain expressions for consumer and supplier welfare with respect to residual demand as follows:

$$\begin{aligned}\mathcal{W}_{\mathrm{D,w}}^{\mathrm{ttl}}(t) = {}& v \min(D^{\mathrm{ttl}}(t) - G_{\mathrm{w}}^{\mathrm{ttl}}(t), D^{\mathrm{ttl}}(t) + R(t) - G_{\mathrm{w}}^{\mathrm{ttl}}(t)) \\ & - c^{\mathrm{bo}} \max(0, -R(t)) \\ & - P(t)(R(t) + D(t) - G_{\mathrm{w}}(t) - r_0^{\mathrm{da}}) \\ & - p^{\mathrm{da}}(t)(d^{\mathrm{da}}(t) - g_{\mathrm{w}}^{\mathrm{da}}(t) + r_0^{\mathrm{da}}) + vG_{\mathrm{w}}^{\mathrm{ttl}}(t) \\[4pt] = {}& v \min(D^{\mathrm{net}}(t), R(t) + D^{\mathrm{net}}(t)) \\ & - c^{\mathrm{bo}} \max(0, -R(t)) - P(t)\big(R(t) + D^{\mathrm{net}}(t)\big) \\ & + (v - p^{\mathrm{da}}(t))d^{\mathrm{da}}(t) + (P(t) - p^{\mathrm{da}}(t))r_0^{\mathrm{da}} \\ & + p^{\mathrm{da}}(t)g_{\mathrm{w}}^{\mathrm{da}}(t) + vG_{\mathrm{w}}(t)\end{aligned}$$

Hence total welfare can be expressed as the sum,

$$\begin{aligned}\mathcal{W}_{\mathrm{D,w}}^{\mathrm{ttl}}(t) = {}& \mathcal{W}_{\mathrm{D,w}}^{\mathrm{rt}}(t) \\ & + \big\{ vd^{\mathrm{da}}(t) - p^{\mathrm{da}}(t)(d^{\mathrm{da}}(t) - g_{\mathrm{w}}^{\mathrm{da}}(t)) \big\} \\ & + \big\{ (P(t) - p^{\mathrm{da}}(t))r_0^{\mathrm{da}} + vG_{\mathrm{w}}(t) \big\}\end{aligned}$$

The first term is just the real-time welfare expression from [13] for the equivalent load D^{net}; the next term in brackets corresponds to the DAM welfare in which the welfare gain $p^{\mathrm{da}}(t)g_{\mathrm{w}}^{\mathrm{da}}(t)$ is due to the use of wind generation. The final term in brackets contains elements that are outside of the control of the consumer under the assumptions of this paper.

For the supplier surplus, the expression is similar to the case without wind:

$$\begin{aligned}\mathcal{W}_{\mathrm{S,w}}^{\mathrm{ttl}}(t) = {}& \big(P(t) - c^{\mathrm{rt}}\big)G(t) + \big(p^{\mathrm{da}}(t) - c^{\mathrm{da}}\big)g^{\mathrm{da}}(t) \\[4pt] = {}& \mathcal{W}_{\mathrm{S,w}}^{\mathrm{rt}}(t) + \big(p^{\mathrm{da}}(t) - c^{\mathrm{da}}\big)g^{\mathrm{da}}(t)\end{aligned}$$

in which $\mathcal{W}_{\mathrm{S,w}}^{\mathrm{rt}}(t)$ is the supplier surplus obtained in the RTM for serving the residual demand $D^{\mathrm{net}}(t)$.

B. Suppliers command the wind

We now consider the other alternative in which wind resources are a part of the supply-side. As in the previous case, we assume that all the available wind generation is dispatched. Hence, the only change is the shifting of the wind dispatch benefits from the consumers to the suppliers. In this case the total social welfare will be unchanged, but the distribution between suppliers and consumers will be

different. From the consumer's viewpoint, following familiar calculations,

$$\begin{aligned}\mathcal{W}_{\mathrm{D,w}}^{\mathrm{ttl}}(t) = {}& v \min(D^{\mathrm{ttl}}(t), G^{\mathrm{ttl}}(t) + G_{\mathrm{w}}^{\mathrm{ttl}}(t)) \\ & - c^{\mathrm{bo}} \max(0, -R(t)) \\ & - P(t)(G(t) + G_{\mathrm{w}}(t)) - p^{\mathrm{da}}(t)(g^{\mathrm{da}}(t) + g_{\mathrm{w}}^{\mathrm{da}}(t)) \\[4pt] = {}& \mathcal{W}_{\mathrm{D,w}}^{\mathrm{rt}}(t) \\ & + \big\{ (v - p^{\mathrm{da}}(t))d^{\mathrm{da}} + (P(t) - p^{\mathrm{da}}(t))r_0^{\mathrm{da}} \big\} \\ & + (v - P(t))G_{\mathrm{w}}(t)\end{aligned}$$

In this case, the only difference due to the wind is given by the last term. The term $vG_{\mathrm{w}}(t)$, as before, has no impact on the discounted mean welfare expression. The supplier welfare now includes terms due to wind generation:

$$\begin{aligned}\mathcal{W}_{\mathrm{S,w}}^{\mathrm{ttl}}(t) = {}& \big(P(t) - c^{\mathrm{rt}}\big)G(t) + P(t)G_{\mathrm{w}}(t) \\ & + \big(p^{\mathrm{da}}(t) - c^{\mathrm{da}}\big)g^{\mathrm{da}}(t) + p^{\mathrm{da}}(t)g_{\mathrm{w}}^{\mathrm{da}}(t) \\[4pt] = {}& \mathcal{W}_{\mathrm{S,w}}^{\mathrm{rt}}(t) \\ & + \big(p^{\mathrm{da}}(t) - c^{\mathrm{da}}\big)g^{\mathrm{da}}(t) + P(t)G_{\mathrm{w}}(t) + p^{\mathrm{da}}(t)g_{\mathrm{w}}^{\mathrm{da}}(t)\end{aligned}$$

Based on this expression we see that the welfare gain $p^{\mathrm{da}}(t)g_{\mathrm{w}}^{\mathrm{da}}(t)$, which in the previous setting was taken by the consumers, is now taken by the suppliers.

Each of these expressions is based on the assumption that all available wind generation will be utilized. Under this assumption, regardless of who commands the wind, there is always a *systemic* impact of wind volatility reflected by the more volatile residual demand. All the terms that can be impacted by wind volatility such as real-time prices and optimal reserve levels are exactly the same no matter who commands the wind. When the wind resources are commanded by suppliers, the benefits quantified by $p^{\mathrm{da}}(t)g_{\mathrm{w}}^{\mathrm{da}}(t)$ go into the suppliers' pocket.

C. Numerical examples

Using the results of the previous section, we can obtain closed-form expressions for the discounted mean welfare of the consumer and the supplier (details can be found in the appendix). We now provide illustrative examples and discuss our results.

The following set of parameters – expressed in *$/MWh* – are used in all our experiments:

$$\begin{aligned} & c^{\mathrm{bo}} = 200,000, \ v = 50 \\ & c^{\mathrm{rt}} = 30, \ c^{\mathrm{da}} = 0.75 * c^{\mathrm{rt}}, \ p^{\mathrm{da}} = 0.85 * c^{\mathrm{rt}}, \end{aligned} \tag{15}$$

resulting in $c^{\mathrm{da}} < p^{\mathrm{da}} < c^{\mathrm{rt}}$. The discount factor is $\gamma = 1/12$ (corresponding to a 12 hour time horizon). We used $\zeta = 200$, the mean demand was taken to be $\bar{D} = 50,000$ MW, and its standard deviation $\sigma = 500$. We view the variance of wind σ_{w}^2 and wind resource penetration as variables and consider a range of their values in the discussion that follows. The coefficient of variation of wind and the percentage of wind resource penetration are denoted, respectively, by

$$c_v := \frac{\sigma_{\mathrm{w}}}{\mathsf{E}[G_{\mathrm{w}}^{\mathrm{ttl}}(t)]} \quad \text{and} \quad k = 100 \frac{\mathsf{E}[G_{\mathrm{w}}^{\mathrm{ttl}}(t)]}{\bar{D}} \tag{16}$$

Fig. 4: Optimal reserves.

Fig. 4 shows the optimal reserves as a percentage of $\bar{D}$, calculated using (8), with respect to c_v and k, and the other parameters held constant. The value of $\bar{r}^*$ in the model without wind generation – indicated as $k = 0$ in the figure – is approximately 10% of $\bar{D}$. This value rises quickly with increased wind penetration or increased variability.

In the following subsections we illustrate the impact of wind generation volatility on the total consumer and supplier welfare under the two different commanding schemes.

1) Consumers command the wind: We show in Fig. 5 the consumer and supplier welfare. It is clear that for a threshold $c_v \sim 0.1$, the impact of wind volatility makes the consumer break even. Below this threshold value, the consumer sees increased benefit with additional wind generation. And beyond it, the consumer welfare decreases rapidly. With high volatility, the consumer is better served by reducing the wind generation injected into the system and, hence, the consumer will not use all the wind generation.
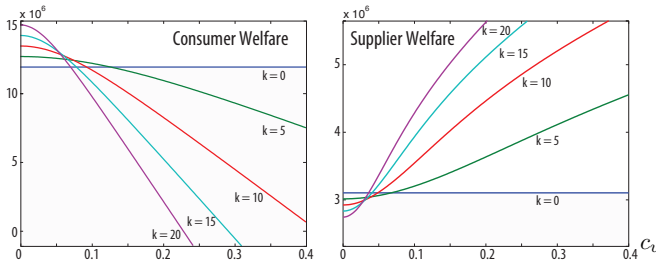

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Fig. 5: Consumer and supplier welfare: consumers command wind.

Remarkably, for the given set of model parameters, the threshold at $c_v = 0.1$ is largely invariant to the level of wind resource penetration.

2) Suppliers command the wind: From the welfare plots in Fig. 6, we see that no matter the volatility, nor the wind penetration level, consumers are always worse off when the supplier commands the wind. Suppliers will benefit from higher volatility levels regardless of who commands the wind. Naturally, their surplus is greater when they are in command of all generating resources.

D. Facing wind volatility

The numerical results illustrate some of the features of multi-settlement energy markets with volatile resources and also invite several questions.

First, there are many ways to address volatility of wind generation: demand management combined with the introduction of storage has been suggested in the literature [14]. Supply management is another approach. In particular, the numerical results of this section illustrate that the current widely-adopted integration policy of 'take all the wind' should be reconsidered. Supply schemes that reduce volatility, even at the cost of reducing the mean energy of the

resource, may play a valuable role in fully exploiting the benefits of wind generation. For example, consider the case of a physically installed wind capacity of 20% shown in Fig. 5. If c_v is between 0.05 to 0.06, then consumers might choose to implement a 75% control rule, in which the wind generation injected into the system is reduced from 20% to 15%. If c_v is larger (say, greater than 0.125 in this example), then the consumers might altogether abandon wind generation or attempt to reduce volatility through truncation of energy above some fraction of forecasted supply.
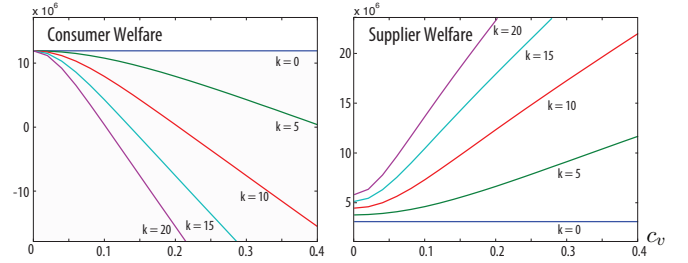

Fig. 6: Consumer and supplier welfare: suppliers command wind.

Next, we consider a critical issue: when treating wind energy as a supply-side resource, why is the consumer exposed to the volatility of wind? An explanation lies in the nature of equilibrium prices in the RTM. Despite unchanged demand, more volatile supply results in a new equilibrium in which the consumers suffer. This is exacerbated in a setting in which all the wind generation is dispatched. However, why the consumer *has* to be exposed to such volatility is unclear.

Alternative schemes to integrate wind generation in which consumers do not have to be exposed to volatility of wind generation outputs, may be favorable due to theoretical and practical considerations or political pressure. Localizing the impact of supply-side volatility is a good example of such a scheme. The ultimate consequences of such policy are not clear. The suppliers of wind generation would suffer from costs, say, through arrangements with other controllable generators or through storage facilities. This disincentive would result in reduced investment in this energy technology.[1]

IV. Concluding Remarks

The main message of this paper is that consumer welfare may fall dramatically as more and more wind generation is dispatched. This is true even under the most ideal circumstances in which the consumer owns all wind generation resources and, *more importantly*, the perfect competition setting in which price manipulation is excluded. Closed-form formulae show that under the current scheme of *dispatch all the wind*, no matter who commands the wind (supplier or consumer), consumer welfare falls and supplier welfare will eventually rise with increases in either wind penetration or its volatility or both.

This work invites many open questions. In addition to those raised in § III-D, we ask: (i) What is the outcome of the complete dynamic game that includes both DAM

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[1]It has been shown in [18] that the level of participation of wind suppliers in forward markets is reduced for large wind volatility.

and RTM? (ii) How do our conclusions change with more realistic representations for cost and utility?[2] Also, how do we analyze the DAM/RTM coupling in an oligopolistic setting? (iii) How can we use these insights to formulate incentives and penalties to improve reliability and encourage participation by private firms? How can we estimate changes in consumer and supplier welfare in a market with more complex policies?

Given the adverse impacts of volatility, we emphasize the need to investigate demand- and supply-side management approaches which can temper such effects. Also, it is critical to minimize the consumer's exposure to volatility of wind resources in order to fully exploit the benefits of their integration.

APPENDIX

In this appendix we derive the formulae for discounted mean social welfare. We begin with some generalities: X is a Markov process on a general state space X, with semigroup $\{P^t : t \geq 0\}$. For a given $\gamma > 0$, the resolvent is the Laplace transform,

$$U_\gamma := \int_0^\infty e^{-\gamma t} P^t \, dt \qquad (17)$$

We let $c \colon \mathsf{X} \to \mathbb{R}$ denote a generic function satisfying $U_\gamma |c|(x) < \infty$ for each x. In this case the function $h = U_\gamma c$ has the representation,

$$h(x) = \mathsf{E}\Big[\int_0^\infty c(X(t)) e^{-\gamma t} dt\Big] \qquad (18)$$

This is part of our motivation for considering the resolvent. The other motivation comes from its relationship with the generator.

A function h is in the domain of the *extended generator* if there exits a function g such that the process below is a *local martingale* for each initial condition of Φ,

$$M_T := h(\Phi_T) - h(\Phi_0) - \int_0^T g(\Phi_s) \, ds, \quad T \geq 0. \qquad (19)$$

We let $\mathcal{A}$ denote the extended generator, and denote $\mathcal{A}f = g$ when M is a local martingale (see [19], [20]).

Under our assumption that $U_\gamma |c|$ is finite-valued, the function $h = U_\gamma c$ is in the domain of the extended generator with

$$\mathcal{A}h = \gamma h - c. \qquad (20)$$

Consequently, the domain of the extended generator includes the range of the resolvent.

Our goal is to compute solutions to dynamic programming equations of the form (20), when the function c is given. The models of interest are limited to two special cases:

(i) $\Phi = R$: The reflected Brownian motion (5) on this domain, so that $\mathsf{X} = (-\infty, \bar{r}]$.
(ii) $\Phi = (R, D)$: where R is in (i), and the demand process D also appears in (5). In this case $\mathsf{X} = (-\infty, \bar{r}] \times \mathbb{R}$.

[2]See [15] for very recent work in this direction

The following result identifies a large class of functions in the domain of $\mathcal{A}$. The result is an interpretation of Îto's formula for reflected diffusions [21].

Proposition 1: Suppose that $h \colon (-\infty, \bar{r}] \to \mathbb{R}$ is C^2 and satisfies $h'(\bar{r}) = 0$. Then h is in the domain of the extended generator, and $\mathcal{A}h = \mathcal{D}h$, where

$$\mathcal{D}h = \zeta h' + \tfrac{1}{2}\sigma^2 h''. \qquad (21)$$

It is often easy to solve the DP equation for $\mathcal{D}$,

$$\zeta h_0'(r) + \tfrac{1}{2}\sigma^2 h_0''(r) = \gamma h_0(r) - c(r), \qquad (22)$$

where the function h_0 is piecewise C^2. In particular, the functions defined below satisfy (22) with $c \equiv 0$:

$$\varphi_+(r) = e^{-\theta_+ r}, \quad \varphi_-(r) = e^{-\theta_- r} \qquad (23)$$

where $\theta_- < 0$ and $\theta_+ > 0$ denote the two roots of the quadratic equation,

$$\tfrac{1}{2}\sigma^2 \theta^2 - \zeta\theta - \gamma = 0. \qquad (24)$$

These functions are building blocks for the solution of (20):

Lemma 1: Suppose that c is a piecewise continuous function, and that h_0 is a piecewise continuous function that is C^2 on each of the intervals $(-\infty, 0]$ and $(0, \infty)$, satisfying (22) for $r \neq 0$. Then the function h defined below is in the domain of the extended generator for R, and satisfies (20):

$$h(r) = h_0(r) + \begin{cases} a_- \varphi_-(r) & r \leq 0 \\ b_- \varphi_-(r) + b_+ \varphi_+(r) & 0 < r \leq \bar{r} \end{cases} \qquad (25)$$

where the constants $\{a_-, b_-, b_+\}$ solve the system of linear equations,

$$\begin{bmatrix} 1 & -1 & -1 \\ \theta_- & -\theta_- & -\theta_+ \\ 0 & \theta_- e^{-\theta_- \bar{r}} & \theta_+ e^{-\theta_+ \bar{r}} \end{bmatrix} \begin{bmatrix} a_- \\ b_- \\ b_+ \end{bmatrix} = \begin{bmatrix} h_0(0+) - h_0(0-) \\ h_0'(0-) - h_0'(0+) \\ h_0'(\bar{r}) \end{bmatrix} \qquad (26)$$

Proof: The matrix equation (26) represents three constraints for the function h: Continuity at the origin, differentiability at the origin, and finally the constraint $h'(\bar{r}) = 0$. While this function is C^1 and not C^2, it can be approximated by C^2 functions to establish the local martingale property. ∎

Lemma 1 is the idea behind the proof of [22, Proposition 3.4.13], which considers the special case in which c is a continuous piecewise linear function. A special case required in the analysis that follows uses,

$$h_0(r) = \gamma^{-1} c(r) \quad \text{when } c(r) = \mathbb{I}\{r \leq 0\}. \qquad (27)$$

In view of (14), to compute the discounted mean welfare functions defined in (9) it is sufficient to solve (20) for the four functions of (r, d),

$$c_A(r, d) = d\mathbb{I}\{r \leq 0\}, \quad c_B(r) = r\mathbb{I}\{r \leq 0\}$$
$$c_C(r) = \mathbb{I}\{r \leq 0\}, \quad c_R(r) = r \qquad (28)$$

We denote by h_A, h_B, h_C, h_R the respective solutions to (20). We then apply (9–14) to obtain,

$$K_{\mathrm{s}}^* = (v + c^{\mathrm{bo}})(h_A + h_B) - ch_R, \quad K_{\mathrm{D}}^* = -(v + c^{\mathrm{bo}})h_A \qquad (29)$$

The functions $\{h_B, h_C\}$ can be computed by a direct application of Lemma 1:

Proposition 2: The function $h_{B0}(r) = (\gamma^{-1}r + \gamma^{-2})\mathbb{I}\{r \leq 0\}$ solves (22) with $c = c_B$. The function $h_{C0}(r) = \gamma^{-1}\mathbb{I}\{r \leq 0\}$ solves (22) with $c = c_C$.

An application of Lemma 1 then gives $h_B = h_{B0} + \varphi_B$ and $h_C = h_{C0} + \varphi_C$, where $\{\varphi_B, \varphi_C\}$ are piecewise continuous, and are linear combinations of $\{\varphi_+, \varphi_-\}$ on the two line segments $(-\infty, 0]$ and $(0, \bar{r}^*]$.

The function h_R is computed in a similar fashion. The proof of Prop 3 follows from Prop 1. The constant k is chosen so that $h'_R(\bar{r}) = 0$.

Proposition 3: $h_R(r) = \gamma^{-1}r + \gamma^{-2} + k\varphi_-(r)$, $r \leq \bar{r}$, with $k = (\theta_- \gamma \varphi_-(\bar{r}))^{-1}$

Computation of h_A is a bit more complex: Let $c_D = h'_C$ (the derivative of h_C), and suppose that h_D is in the domain of the extended generator and solves (20) with $c = c_D$. We then have

Proposition 4: $h_A(r, d) = dh_C(r) - \sigma^2 h_D(r)$

The proof of Prop 4 follows from the following lemma.

Lemma 2: The function H_1 defined by $H_1(r, d) = dh_C(r)$ is in the domain of the extended generator for the bivariate process $(\boldsymbol{R}, \boldsymbol{D})$, and satisfies,

$$\mathcal{A}H_1(r, d) = \gamma H_1(r, d) - c_A(r, d) + \sigma^2 c_D(r) \qquad (30)$$

Computation of each of the welfare functions is possible once we can compute h_D. For this we note that c_D can be expressed,

$$c_D(r) = \begin{cases} A_- e^{-\theta_- r} & r \leq 0 \\ B_- e^{-\theta_- r} + B_+ e^{-\theta_+ r} & 0 < r \leq \bar{r}^* \end{cases} \qquad (31)$$

where A_-, B_-, B_+ are constants. Computation of h_D then follows from Lemma 1 combined with the following result:

Lemma 3: Writing $c(r) = c_D(r)$, the function h_0 given below solves (22):

$$h_0(r) = \begin{cases} (\theta_- \sigma^2 - \zeta)^{-1} A_- r e^{-\theta_- r} & r \leq 0 \\ (\theta_- \sigma^2 - \zeta)^{-1} B_- r e^{-\theta_- r} \\ \quad + (\theta_+ \sigma^2 - \zeta)^{-1} B_+ r e^{-\theta_+ r} & 0 < r \leq \bar{r}^* \end{cases}$$

Proof: We apply the product rule $\mathcal{D}fg = f\mathcal{D}g + g\mathcal{D}f + \sigma^2 f'g'$, using $f(r) = r$ and $g(r) = e^{-\theta r}$, where θ is any solution to (24). This gives,

$$\mathcal{D}fg(r) = r\gamma e^{-\theta r} + \zeta e^{-\theta r} - \theta \sigma^2 e^{-\theta r}$$

Consequently, considering the two possibilities for θ, on defining $h_+ = r\varphi_+/(\theta_+ \sigma^2 - \zeta)$, and $h_- = r\varphi_-/(\theta_+ \sigma^2 - \zeta)$, we obtain,

$$\mathcal{D}h_+ = \gamma h_+ - \varphi_+ \quad and \quad \mathcal{D}h_- = \gamma h_- - \varphi_-$$

From the formula (31) we conclude that $\mathcal{D}h_0 = \gamma h_0 - c_D$, with h_0 defined in the lemma. ∎

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