A Coordinated Route-Guidance System For Connected Vehicles Under Mixed-Strategy Congestion Game With Information Perturbation (CRM-M-IP)

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Game Theory and Control Mechanisms in Transportation Operations Session
Overview

1. Motivation
   a. Coordinated Routing
   b. Information Perturbation
2. CRM-M-IP Model
3. Analytical Results
4. Numerical Results
5. Future Work
Motivation – Coordinated Routing

• When drivers use route guidance tools make routing decisions *independently*, problems can occur
  • Overreaction, flash-crowd effect

• **Coordinated routing**: routing based on current traffic conditions *and* tentative route choices of others using route guidance
Motivation – Information Perturbation

• Drivers typically make route choices selfishly in an effort to minimize personal travel time

• From a system perspective, system performance can be improved under different route-choice behavior

• Information perturbation: strategically modify traffic information available to users to ensure a certain level of system performance
Combining those two:

• Leverage connected vehicle technology (V2V, V2I) to create an online *coordinated* routing scheme
  • Account for uncertainty in decision making

• While integrating strategic *information perturbation*, aiming to improve system performance without over-sacrificing user optimality

**CRM-M-IP**
Equilibrium Routing Decision (ERD)
Mixed Strategy – probability distribution over candidate paths

“System-optimal”

SUDA (Du et al. 2014)
MP for Mixed-Strategy ERD

MP – ERD: \( \min Z(\tilde{f}) = \int_{0}^{\tilde{f}_l} \tilde{C}_{l,\lambda}(\omega) \, d\omega + \sum_{v=1}^{M} \sum_{i=1}^{k_v} \frac{1}{\gamma_v} \tilde{f}_{v,i} \ln(\tilde{f}_{v,i}) \)

s.t. \( \sum_{i=1}^{m} \tilde{f}_{v,i} = 1, \forall v \)

\( \tilde{f}_{l} = \sum_{v=1}^{M} \sum_{i=1}^{k_v} \tilde{f}_{v,i} \delta_{v}^{l,i} \quad \forall l \)

\( \tilde{f}_{v,i} \geq 0, \forall v, i \)

Perturbed cost function

Logit function

Heterogeneous users

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CRM-IP-M
Perturbed cost function

- Average travel time: $C_l(f_l)$
  - User optimal routing
- Marginal travel time: $\hat{C}_l(f_l) = f_lC'_l(f_l) + C_l(f_l)$
  - System optimal routing
- Perturbed cost function: $\tilde{C}_{l,\lambda}(f_l) = \lambda\hat{C}_l(f_l) + (1 - \lambda)C_l(f_l)$, $\lambda \in [0,1]$
CRM-M-IP Objectives

Benefits of CRM-M: Du et al. 2014

**CRM-M-IP:**

Explore how information perturbation impacts:

- Change of the potential function $\Delta_\lambda^P$
- Loss of user optimality $\Delta_\lambda^U$
- Gain of system performance $\Delta_\lambda^S$

1. Analytically
2. Simulation

How are they related?
Change of the potential cost function \((\Delta^P_\lambda)\)

- Unperturbed MP-ERD: \(Z(f, C)\)
- Perturbed MP-ERD: \(Z(\tilde{f}, C)\)

\[ \Delta^P_\lambda = Z(\tilde{f}, C) - Z(f, C) \]
Loss of user optimality \( (\Delta^U_\lambda) \)

User Optimality:

\[
Z_u(\tilde{f}_{v,i}) = \sum_{i=1}^{k_v} \tilde{f}_{v,i} \Gamma_v(i) (\tilde{f}_{v,i})
\]

Where: \( \Gamma^i_v(\tilde{f}_{v,i}) = \) perceived travel cost

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Loss of user optimality ($\Delta_U^\lambda$)

$f_v^*$: flow preference for vehicle $v$ that reduces expected travel cost
$ar{f}_v$: flow preference for vehicle $v$ that minimizes expected travel cost

$$\text{MP} - \text{UO} = \min Z_u(\bar{f}_v) = \sum_{i=1}^{k_v} \bar{f}_{v,i} \left( \sum_{l \in h_{v,i}} C_l(\bar{f}_l) + \frac{1}{\gamma_v} \ln(\bar{f}_{v,i}) \right)$$

s.t. $\sum_{i=1}^{k_v} \bar{f}_{v,i} = 1$

$f_v^i \geq 0$
Loss of user optimality \( (\Delta^U_\lambda) \)

\[
\Delta^U_\lambda = \sum_{i=1}^{k_v} \tilde{f}_{v,i} \Gamma_{v,i}(\tilde{f}_{v,i}) - \sum_{i=1}^{k_v} f_{v,i} \Gamma_{v,i}(f_{v,i})
\]
Gain of system performance \( (\Delta^S_{\lambda}) \)

\[
\text{MP} - \text{SP}: \min S(f) = \sum_{i \in A} f_i \ C_i(f_i) + \sum_{v=1}^{M} \sum_{i=1}^{k_v} \frac{1}{\gamma_v} f_{v,i} \ln(f_{v,i})
\]

Yang (1999)

Unperturbed Mixed-Strategy SP : \( S(f, C) \)

Perturbed Mixed-Strategy SP : \( S(\tilde{f}, C) \)

\[
\Delta^S_{\lambda} = S(f, C) - S(\tilde{f}, C)
\]

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Explore the relationship between $\Delta_\lambda^P, \Delta_\lambda^U, \Delta_\lambda^S$

$$\Delta_\lambda^S \geq \frac{(1 - \lambda)}{\lambda} \Delta_\lambda^P$$

Lemma 1: The gain of system optimality due to information perturbation is greater than or equal to the corresponding change in potential cost times some constant related to the perturbation amount.
Explore the relationship between $\Delta^P_\lambda, \Delta^U_\lambda, \Delta^S_\lambda$

$\Delta^U_\lambda \leq \Delta^P_\lambda$

Lemma 2: The loss in user optimality due to information perturbation is no greater than the change in the potential cost.
Analytical Results: $\Delta^{P}_{\lambda}, \Delta^{U}_{\lambda}, \Delta^{S}_{\lambda}$

\[
\Delta^{S}_{\lambda} \geq \frac{(1 - \lambda)}{\lambda} \Delta^{P}_{\lambda} \\
\Delta^{U}_{\lambda} \leq \Delta^{P}_{\lambda}
\]

Implication: A small perturbation amount leads to major system improvement with relatively low user loss

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Numerical Experiments

• Explore impacts of perturbation for different LOS (v/c ratio)
  • System performance
  • Impacts on users
• MATLAB
• SUDA (Du et al. 2014)
Improvements in system performance

MP-SP Improvement (%) vs. $\lambda$

True SP Improvement (%) vs. $\lambda$

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User optimality sacrifice

Average UO Sacrifice (%) vs. $\lambda$

Maximum UO Sacrifice (%) vs. $\lambda$

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Impacts on travel time

Users with Travel Time Reduction (%) vs. $\lambda$

Travel Time Reduction (%) vs. $\lambda$

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Summary

1. CRM-M-IP Model
2. Analytical Results
   a. Small perturbation amounts lead to significant system performance gains with relatively low user loss
3. Numerical Results
   a. System performance improvement >4%, individual travel time savings >5% in congested scenarios
   b. Significant user loss in high perturbation scenarios
Future work

• Use simulation output to build application-friendly models
  • Optimal perturbation, ensuring user compliance
• Selective perturbation
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Thank you!

Questions?

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