Modeling Alongshore Currents over Alongshore-Uniform Barred Beaches

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Introduction

The nonlinear dynamics of alongshore currents in the surf zone over an alongshore uniform barred beach topography are studied using numerical experiments. The purpose of the work is to improve the existing numerical model predictive capabilities by considering the effects of the following:

- wave-current interaction
- linear or non-linear bottom friction
- a roller model
- data assimilation

The numerical predictions are compared to field data collected during the Delilah experiment in October, 1990, over a barred beach at Duck, North Carolina.

Formulation

The two-dimensional nonlinear shallow water equations are used to describe the depth-averaged and time-averaged (with respect to the period of the waves) mean flows:

$$(hu)_x + (hv)_y = 0$$

$$u_t + uu_x + vu_y = \frac{p_x}{\rho} + \tau_x^w - \frac{C_f < |\vec{u}| u >}{h} - \nu\nabla^4 u$$

$$v_t + uv_x + vv_y = \frac{p_y}{\rho} + \tau_y^w - \frac{C_f < |\vec{u}| v >}{h} - \nu\nabla^4 v$$

- x is in the cross-shore direction
- y is in the alongshore direction
- wave forcing effects are approximated τ_i^w by a radiation stress fomulation
- bottom friction included in C_f term
- ν is a small biharmonic diffusion coefficient used to increase the numerical stability

The wave forcing effects are modeled using a differential equation for the wave energy, E_w

$$\left[\frac{E_w}{\sigma}\right]_t + \left[(u + C_{g1})\frac{E_w}{\sigma}\right]_x + \left[(v + C_{g2})\frac{E_w}{\sigma}\right]_y = -\frac{\varepsilon_w}{\sigma}$$

where C_{gi} is the group velocity, σ is the intrinsic angular frequency, and ε_w is the dissipation rate of wave energy which is obtained from the Church and Thornton (1993) wave-breaking model.

When wave-current interaction is employed, the E_w equation is coupled with the shallow water equations and integrated in time and space.

When wave-current interaction is not employed, E_w is evaluated using the initial flow conditions only, which have zero velocity everywhere.

Mean wave height predictions are reasonably accurate.

Wave- Current Interaction

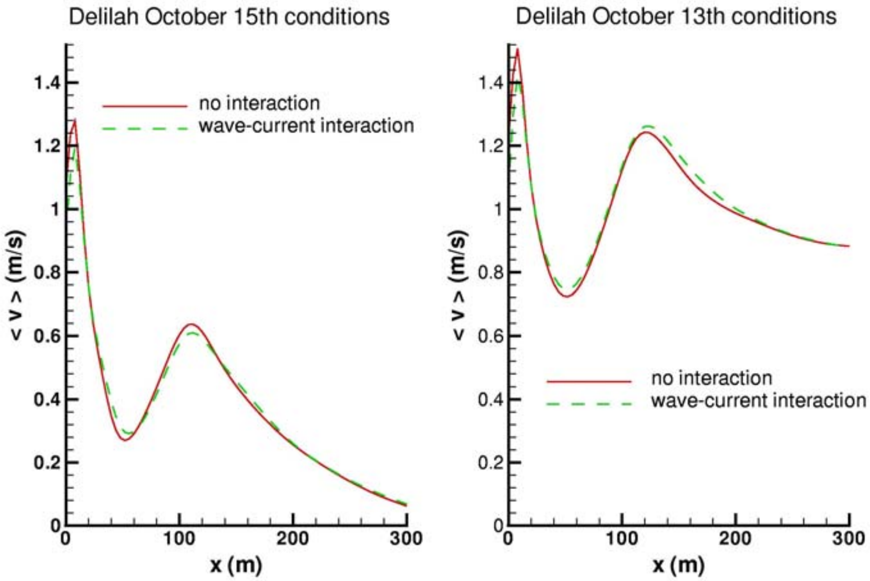

Fig. 1: Time- and alongshore-averaged alongshore current predictions with and without wave-current (w-c) interaction

- Wave-current interaction has little effect on the results.
- Nevertheless, wave-current interaction is implemented henceforth.

Bottom Friction

- linear: $C_f < |\vec{u}| u_i > = \mu u_i$
- nonlinear: $< |\vec{u}| u_i > = \int_T \vec{u}\, u_i \, dt$
- Wright and Thompson (1983) non-linear correlation (WT):

$$< |\vec{u}| u_i > = \sigma_T u_i \sqrt{\alpha^2 + \left(\frac{u_i}{\sigma_T}\right)^2}$$

where α is a constant and σ_T^2 is the total velocity variance.
C_f can be constant, or can vary with the bed roughness k_a (Ruessink *et al*., 2001):

$$C_f = 0.015\left(\frac{k_a}{h}\right)^{1/3}$$

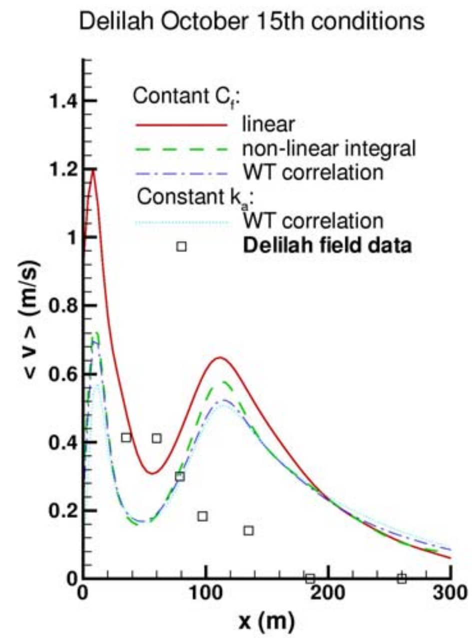

Fig. 2: Time- and alongshore-averaged alongshore currents with different bottom friction parameterizations

- All parameterizations predict an artificial peak at x = 20m because the wave energy is forced to zero at the shoreline.
- The predicted alongshore current peak at x = 120m is located too far offshore, compared to the field data.
- Non-linear parameterizations have the greatest effect near the shore.
- The WT correlation runs twice as fast as the integral parameterization.

Roller Model

- Shifts momentum input from breaking waves to the water column and alongshore current predictions shoreward
- Modeled in 2D with a time dependent p.d.e. for the roller energy E_r
- Dissipated wave energy ε_w is stored as roller energy, and shoals with the wave field before being dissipated, the net (wave + roller) radiation stresses gradients produce mean along shore currents closer to the shore
- Roller dissipation, $\varepsilon_r = \tau_r C = \rho g A \sin\sigma / T$, is a key quantity.
- Results depend on the roller area. Three area formulations are tested:

    1. Engelund/Lippmann *et al.* (1996): $A = H_b^3 / 4h \tan\sigma$
    2. Svendesen (1984): $A = 0.9\,H$
    3. New $A = C_1 E_r$ with linear damping near the shore ($0 < x < 25$ m); the constant C_1 is set to 0.75 to give results similar to the other cases
    4. An "ideal" area, derived mathematically from a cubic curve fit of the velocity field data (used for comparison)

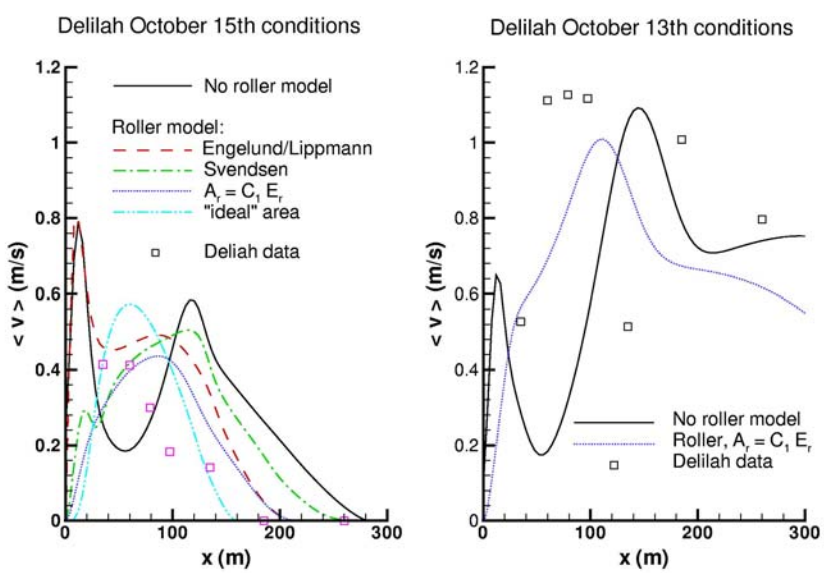

Fig. 3: Time- and alongshore-current predictions using the roller model

- A roller model can shift the alongshore current showerwards
- A roller model can eliminate the artificial twin peak at $x = 20$m
- Choice of the roller area affects the performance
- The "ideal" roller area gives best predictions
- Best physical roller area is based on the roller energy, $A = C_1 E_r$

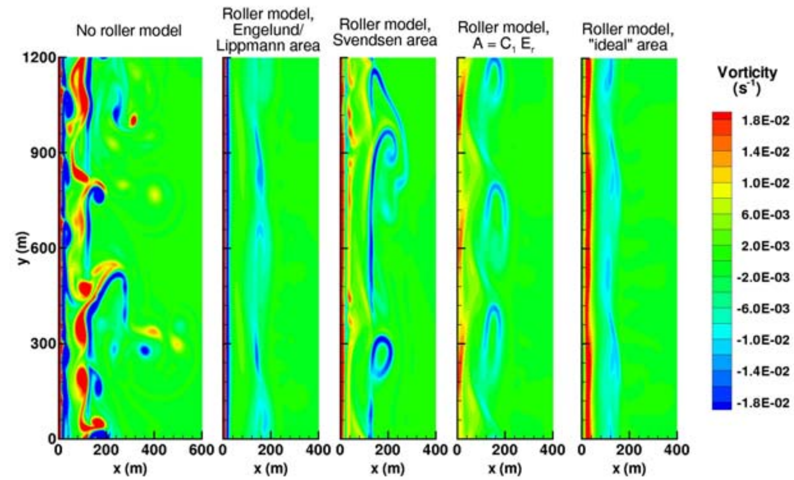

Fig. 4: Vorticity predictions at an instant in time using the roller model with the October 15th Delilah conditions

- Mean and instantaneous vorticity patterns in the flow are adjusted
- Mean current instabilities are damped as the alongshore current profile is shifted towards shallower water and contains smaller cross-shore gradients
- Adding a roller model affects the modeled alongshore current instabilities in the flow and subsequent cross-shore mixing by non-linear shear instabilities
- In some formulations, not all of the roller energy needs to be dissipated by the time the wave reaches the shore line

Data Assimilation

- Used to produce good approximations to field data with a non-ideal model
- Requires some field data *a priori*
- Commonly used to model large fluid bodies (*e.g.* atmosphere, oceans); has not been used for 2D nearshore circulation modeling before
- Solution is nudged towards desired solution each timestep using optimal interpolation of the field data

$$\vec{v}_{new} = n\,\vec{v}_{oi} + (1-n)\vec{v}_{old}$$

where $_{oi}$ refers to optimal interpolation and n is a percent value between 0 and 100.

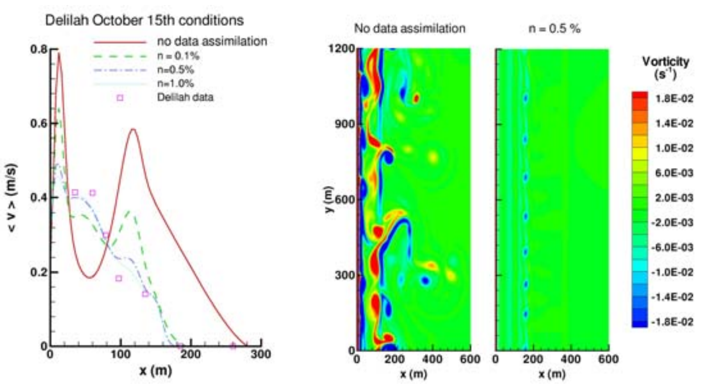

Fig. 5: Time- and alongshore-averaged alongshore currents, and instantaneous vorticity predicted using data assimilation. The code is run with the WT bottom friction correlation (C_f = constant) and without a roller model.

- A small amount of optimal interpolation will force the solution to the desired field data
- Data assimilation also dampens the alongshore instantaneous vortices, as observed with the roller model simulations

Summary

- Wave-current interaction does not have a great impact on the results for an alongshore uniform beach

- A non-linear bottom friction parameterization is important very close to the shore

- A roller model with an appropriate roller area greatly improves the model predictions

- New roller area based on the roller energy $A = C_1 E_r$ worked well when linear damping was used between $0 < x < 25$ m

- Data assimilation can also be used to improve predictions made with a non-ideal model