1. The properties of a norm are:

(a) $N(x) \geq 0$ and $N(x) = 0$ if and only if $x = 0$.
(b) $N(ax) = |a|N(x)$.
(c) $N(x + y) \leq N(x) + N(y)$

Since $A$ is positive definite, it has full rank so $x^tAx \geq 0$ and $x^tAx = 0$ if and only if $x = 0$ so the first property holds.

If $a \in \mathbb{R}$, then $(ax)^tA(ax) = a^2(x^tAx) \neq |a|(x^tAx)$ unless $a = 0$. Therefore, the second property does not hold so it is not a norm. NOTE: This is the only part of the answer that is required.

2. $\|x - y\|^2 = \|x\|^2 + \|y\|^2 - 2x^ty = 2 - 2\cos(\theta)$.

Since $\theta = \frac{\pi}{3}$, the relationship implies that $\|x - y\|^2 = 2 - 2\cos\left(\frac{\pi}{3}\right) = 1$.

On the other hand, direct calculation shows that $\|x - y\|^2 = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$.

3. (a) Diagonalizing a matrix $A$ means finding an invertible matrix $S$ and a diagonal matrix $\Lambda$ such that $A = SAS^{-1}$.
(b) $C$ is symmetric because $C^t = (U\Lambda U^t)^t = (U^t)^t\Lambda^tU^t = U\Lambda U^t = C$.
(c) $\Lambda^{-\frac{1}{2}}U^tCU\Lambda^{-\frac{1}{2}} = \Lambda^{-\frac{1}{2}}A\Lambda^{-\frac{1}{2}} = I$.

4. The problem requires calculating $c_1$ and $c_2$ such that $x = c_1v_1 + c_2v_2 = Sc$ where the columns of $S$ are $v_1$ and $v_2$. This requires solving a linear system using a sequence of calculations such as elimination of variables by taking linear combinations of rows of $S$. It is conceptually equivalent to calculating $S^{-1}$ and then $S^{-1}x$ to get $c$. The solution is $S^{-1}c = \left(\frac{5}{8}, \frac{3}{8}\right)^t$.

5. This problem is much easier because $S^{-1} = S^t$ since $v_1$ and $v_2$ are orthonormal. Therefore, no arithmetic operations are required to compute $S^{-1}$. The same fact can be seen as a consequence of the orthonormality of $v_1$ and $v_2$ by noting that $x = c_1v_1 + c_2v_2 \implies c_k = v_k^t x$ for $k = 1, 2$. 