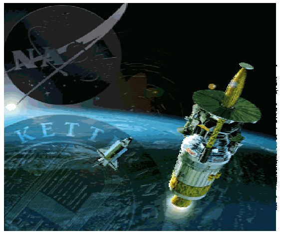
Finite Element Based Hydrodynamic Sheath Model

Subrata Roy and Birendra P. Pandey

Computational Plasma Dynamics Laboratory

Kettering University, Flint, MI 48504

http://cpdl.kettering.edu



Presented by Prof. Joseph Shang

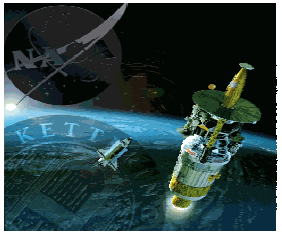
AIAA-2002-2169

Problem of Interest

- 1D formulation of collisional plasma-sheath
  - High-power in-space electric propulsion systems.
    - MPD, SPT type thrusters.
  - High speed air vehicles.
    - Electromagnetic flow control.
  - Magnetically confined fusion plasmas.
    - Tokamak.
  - Material processing in micro-electronics.
    - Thin film deposition, plasma etching.

Challenges

- Very high property gradients.
- Better resistivity models.
- Ionization and recombination processes.
- Accurately calculating fall voltage and energy losses.
- Predicting sheath-presheath boundary.
- Understanding unsteady edge instability (RT modes?).
- Improving flow control.

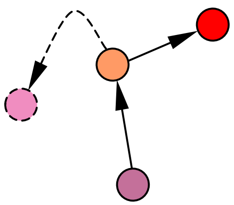


Collision Model

Ionization

$$S_{ioniz} = n_e n_n \left\langle V_{eth} \sigma_i (V_{eth}) \right\rangle = k_i n_e n_n$$

$$k_i = \left[k_i^{0+}, k_i^{0++}, k_i^{1++} \right]$$
$$= \left(-3.2087 \times 10^{-5} \; T_e^3 - 0.0022 \; T_e^2 + 0.7101 \; T_e - 1.76 \right) \times 10^{-14}$$



$$Xe_{\mathrm{fast}}^{+} + Xe_{\mathrm{slow}}^{0} \rightarrow Xe_{\mathrm{slow}}^{+} + Xe_{\mathrm{fast}}^{0}$$
$$Xe_{\mathrm{fast}}^{++} + Xe_{\mathrm{slow}}^{0} \rightarrow Xe_{\mathrm{slow}}^{++} + Xe_{\mathrm{fast}}^{0}$$
$$Xe_{\mathrm{fast}}^{++} + Xe_{\mathrm{slow}}^{0} \rightarrow Xe_{\mathrm{slow}}^{+} + Xe_{\mathrm{fast}}^{+}$$

Recombination

$$S_{recom} = -n_e n_i \left\langle V_{eth} \sigma_{ei}^{r} (V_{eth}) \right\rangle = -\alpha n_e n_i$$

$$\alpha = 1.09 \times 10^{-20} \; n_e \; T_e^{-9/2} \; \mathrm{m}^3/\mathrm{s}$$

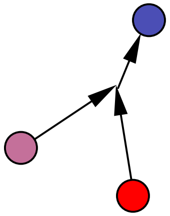
Charge exchange collision cross - section :

$$\sigma_c (Xe - Xe^{+}) = (142.21 - 23.20 \log_{10} \Delta V) \times 10^{-20} \, \mathrm{m}^2$$

Electron - neutral collision cross - section :

$$\sigma_{en} (Xe - Xe^{+}) \approx 27 \times 10^{-20} \, \mathrm{m}^2$$

Ion - neutral collision frequency : $\nu_{in} << \nu_{en}$ as $V_{ith} << V_{eth}$



Electron-ion collision:

$$\nu_{ei} = \frac{4\sqrt{2\pi}}{3\sqrt{m_e}} \frac{e^4 n_i L_e}{T_e^{3/2}} = \frac{L_e}{3\sqrt{2\pi}} \left(\frac{n_i}{n_e} \right) \left(\frac{\omega_{pe}}{n_e \lambda_{De}^3} \right)$$

Plasma-neutral collision:

$$\nu_{en} = n_n < \sigma \, V_{eth} >$$

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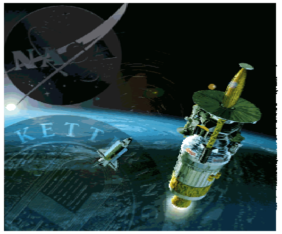
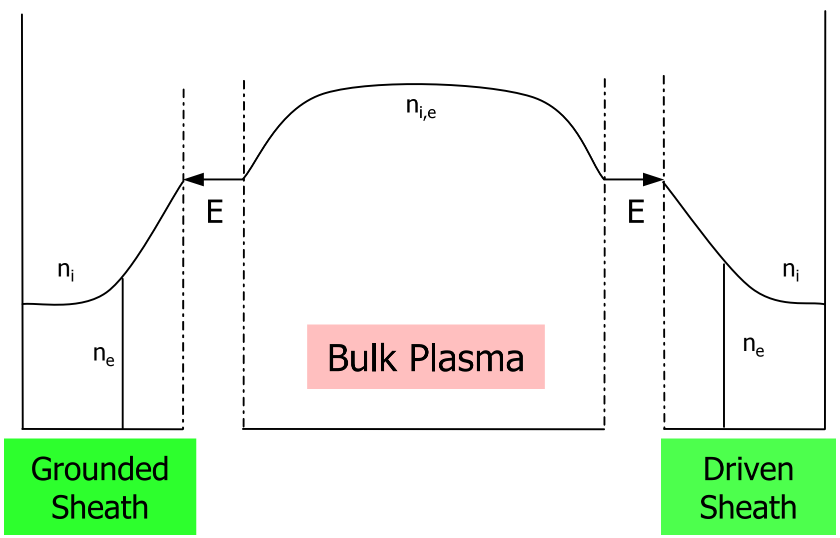
Sheath Theory

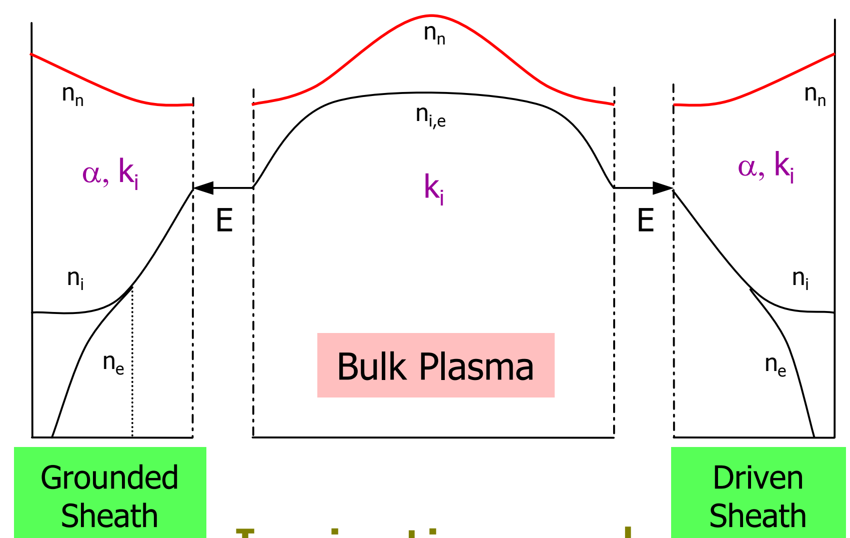


Godyak-Sternberg (1990)
Nitschke-Graves (1995)

Our model



No ionization



Ionization and recombination

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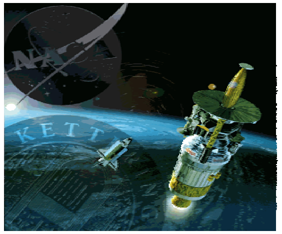

Sheath Criteria

$$\frac{\text{Characteristic ambipolar ion drift speed}}{\text{Bohm velocity}} = \sqrt{\frac{n_i}{n_e}} > 1 \Rightarrow$$

Sheath formation at a lower ion drift speed than Bohm velocity

Modified Bohm Velocity

$$V_B = \sqrt{\frac{2\ (T_e + T_i)}{m_i}} \left(1 + \frac{\pi\lambda_D}{2\lambda_i}\right)^{-1/2}$$

Electron Debye Length

$$\lambda_D = \sqrt{\frac{\varepsilon_0 T_e}{e^2 n_e}}$$

Effective Ion Mean Free Path

$$\lambda_i = \frac{V_B}{\nu_{in} + \nu_{ie} + \nu_c}$$

Pre-sheath thickness ~ λ_{mfp} plasma neutral interaction

Sheath thickness ~ λ_{D} Debye length

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Governing Equations

Continuity:

$$\begin{cases} \dfrac{\partial n_\alpha}{\partial t} + \dfrac{\partial (n_\alpha V_\alpha)}{\partial z} = S_{ioniz} - S_{recomb}, \quad \text{for}\, \alpha = e, i \\ \dfrac{\partial n_n}{\partial t} + \dfrac{\partial (n_n V_n)}{\partial z} = S_{recomb} - k_i^{0+} n_e n_n - k_i^{0++} n_e n_n \end{cases}$$

Momentum:

Ion

$$\frac{\partial V_i}{\partial t} + V_i \frac{\partial V_i}{\partial z} = -\frac{T_i}{m_i n_i}\frac{\partial n_i}{\partial z} - \left(\frac{Ze}{m_i}\right)\frac{\partial \varphi}{\partial z} - \nu_c V_i + \left(\frac{m_e}{m_i}\right)\nu_{ei}\left(V_e - V_i\right)$$
$$-0.5\nu_{in}\left(V_i - V_n\right) - \left(S_{ioniz} - S_{recomb}\right)\frac{V_i}{n_i}$$

Electron

$$\frac{\partial V_e}{\partial t} + V_e \frac{\partial V_e}{\partial z} = -\frac{1}{m_e n_e}\frac{\partial p_e}{\partial z} - \left(\frac{e}{m_e}\right)\frac{\partial \varphi}{\partial z} - \nu_{ei}\left(V_e - V_i\right) - \left(S_{ioniz} - S_{recomb}\right)\frac{V_e}{n_e}$$

$$V_n = \text{const.}$$
$$T_i << T_e$$
$$T_n << T_e$$

Electron Energy:

$$\frac{3}{2}\left(\frac{\partial T_e}{\partial t} + V_e \frac{\partial T_e}{\partial z}\right) = -T_e \frac{\partial V_e}{\partial z} + \nu_{ei}\left(V_e - V_i\right)^2 - \left(\frac{S_{ioniz} - S_{recomb}}{n_e}\right)\left(\frac{3}{2}T_e - E_i\right)$$

Potential:

$$\varepsilon_0 \nabla^2 \varphi = -e\left(n_i - n_e\right)$$

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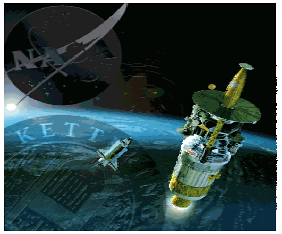
Finite Element Based Hydrodynamic Model

- Developed by the Computational Plasma Dynamics Laboratory at Kettering University.
- A family of complex geometry subroutines that can study macroscopic collisional plasmas.
- Written in Fortran 77, use Cray-style Fortran pointers, and are designed for UNIX-type environment.
- Two and half dimensional formulation (so far).
- Implemented Sub-Grid eMbedded (SGM) FE for Coarse-grid Solution Stability, Accuracy and Tri-diagonal Efficiency.
- Utilized to model low pressure Hall (SPT) and MPD thruster applications.



Numerical Details

Weak Statement

$$L(\mathbf{U})=0;\ \text{where } \mathbf{U}=\{n_e,n_i,n_n,V_e,V_i,T_e,\varphi\}^{\mathrm{T}}$$

$$\int_{\Omega} w\,L(\mathbf{U})\,d\Omega=0,\ \ w \text{ is any admissible test function.}$$

Discrete Approximation

$$\Omega^h=\bigcup_{el}\Omega_{el}\ ;\ u(t,x_j)\approx u^h(t,x_j)=\bigcup_{el}u_{el}(t,x_j);\text{ and } u_{el}(t,x_j)=N_k(x_j)\mathrm{U}_{el}(t)$$

N_k is appropriate basis function; Chebyshev, Lagrange or Hermite interpolation polynomials complete to degree k.

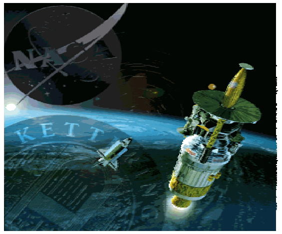
FE Formulation

$$\Omega^h=\bigcup_{el}\Omega_{el}\ ;\ \mathrm{WS}^h=S_{el}\left(\int_{\Omega_{el}}N_k L(\mathbf{U}_{el})\,d\tau\right)\ \Rightarrow\ \mathbf{M}\frac{d\mathbf{U}}{dt}+\mathbf{R}(\mathbf{U})=0;\ \mathbf{M}=S_{el}(\mathbf{M}_{el})$$

Sub-Grid eMbedded FE

$$\int_{\Omega_{el}}\frac{\partial N_{S=1}}{\partial x_j}\frac{\partial N^{\mathrm{T}}_{S=1}}{\partial x_j}\,d\tau=\left[\int_{\Omega_{el}}g(h_j,V_j)\frac{\partial N_{k=2}}{\partial x_j}\frac{\partial N^{\mathrm{T}}_{k=2}}{\partial x_j}\,d\tau\right]^R\ \Rightarrow\ \mathbf{M}\frac{d\mathbf{U}}{dt}+\mathbf{R}_S(\mathbf{U})=0$$

Roy and Baker (1997,1998)



Solution Procedure

NR Iteration

$$\mathbf{U}_{\tau+1}^{i+1} = \mathbf{U}_{\tau+1}^{i} + \Delta\mathbf{U}^{i} = \mathbf{U}_{\tau} + \sum_{p=0}^{i} \mathbf{U}^{p+1}, \text{ where}$$

$$\Delta\mathbf{U}^{i} = -\left[\mathbf{M} + \vartheta\Delta t(\partial\mathbf{R}/\partial\mathbf{U})\right]^{-1} \mathbf{R}(\mathbf{U})$$

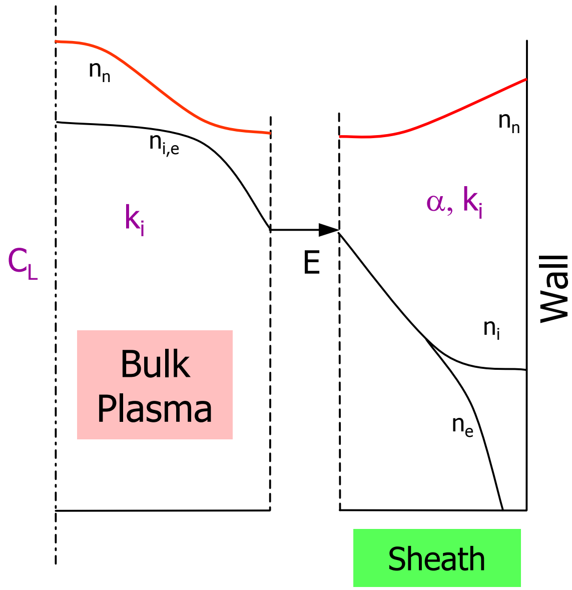
Convergence Criteria

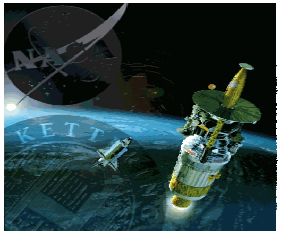
$$\boxed{\frac{\|\mathbf{U}_{j} - \mathbf{U}_{j-1}\|}{\|\mathbf{U}_{j}\|} \leq \in} = 10^{-4} \text{ for all integrated quantities.}$$

- Steady state takes ~100 microsecond.
  Average timestep 50 nanosecond (2,000 steps).

Boundary Conditions

- Zero Plasma Velocity and Finite Plasma Density Imposed at C_L.
- Finite Neutral Density Imposed at C_L.
- Wall Maintained at an Imposed Negative Potential.
- Fixed Electron Temperature at C_L.
- Homogeneous Neumann (Zero Flux) Condition at all other Boundaries.
- ✓ Ion Drift Speed = Modified Bohm Velocity at the Plasma-Sheath Interface.
- ✓ Imposed Electric Field at the Plasma-Sheath Boundary.





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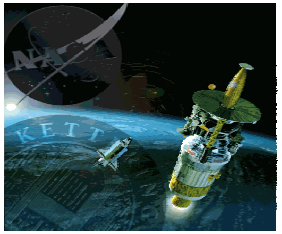
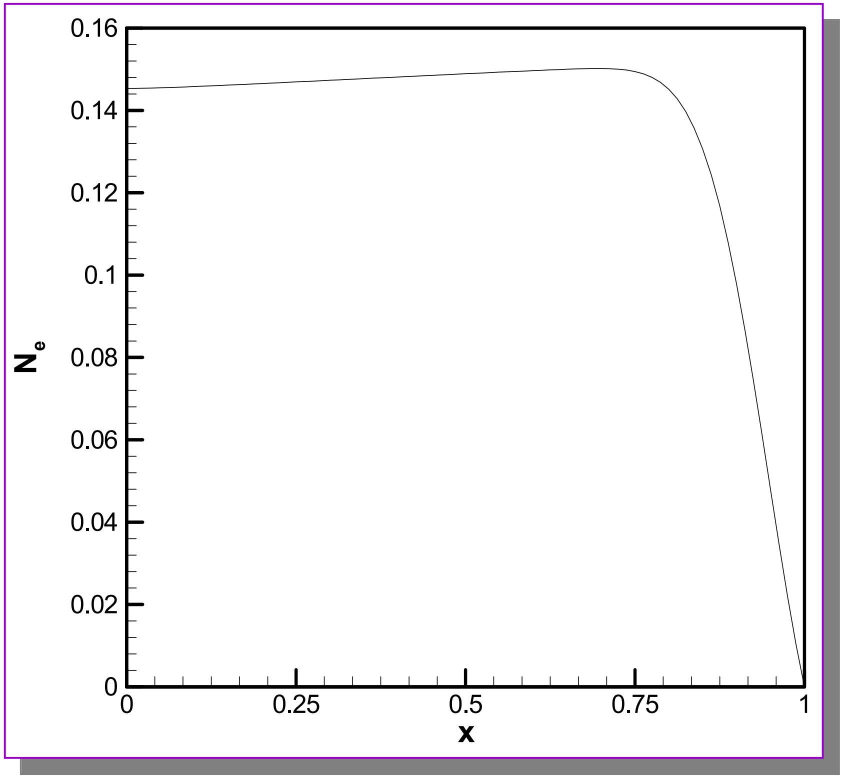
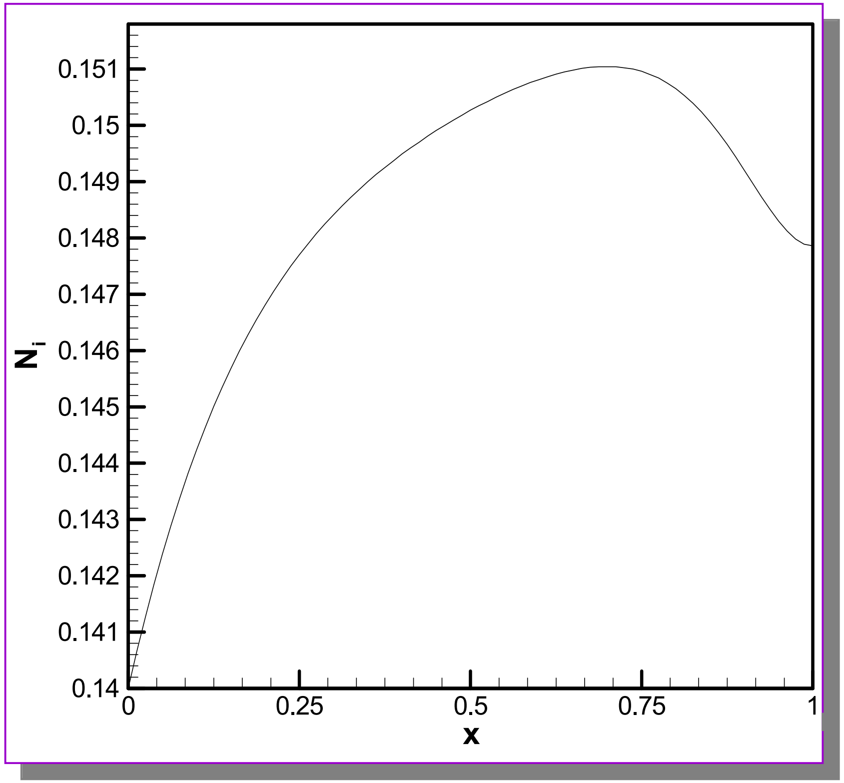
Steady State Solutions
(Number Densities)



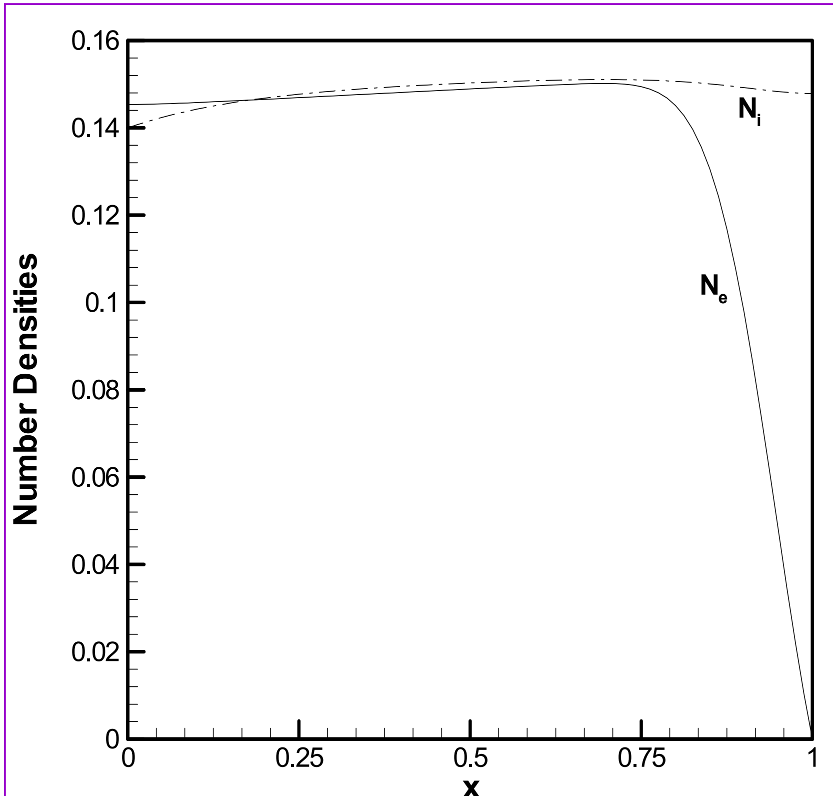


Electron Number Density

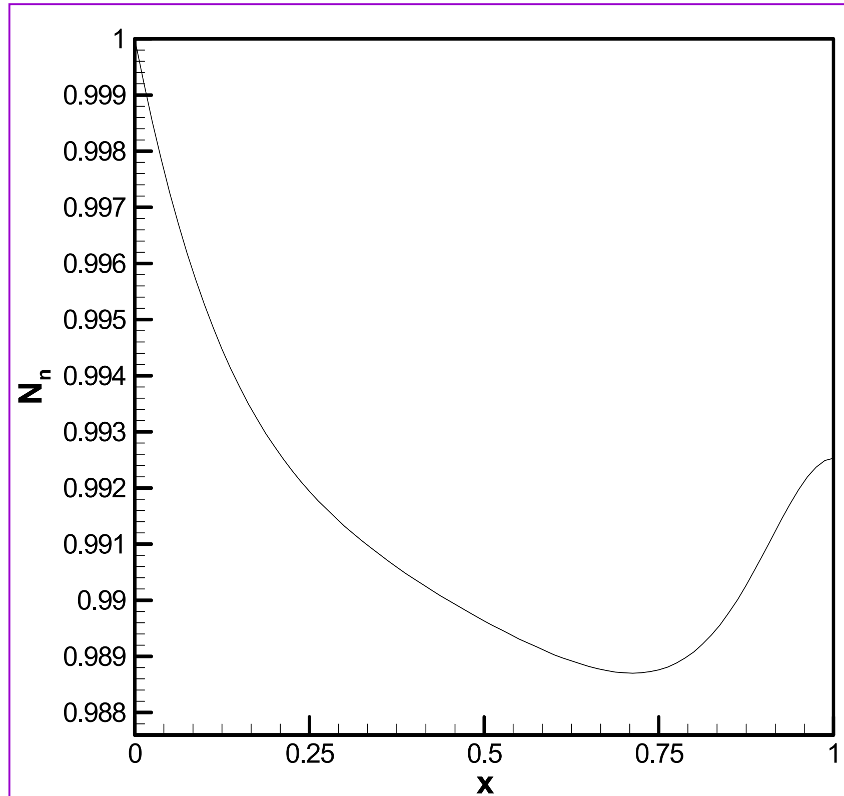


Ion Number Density

Steady State Solutions
(Number Densities)

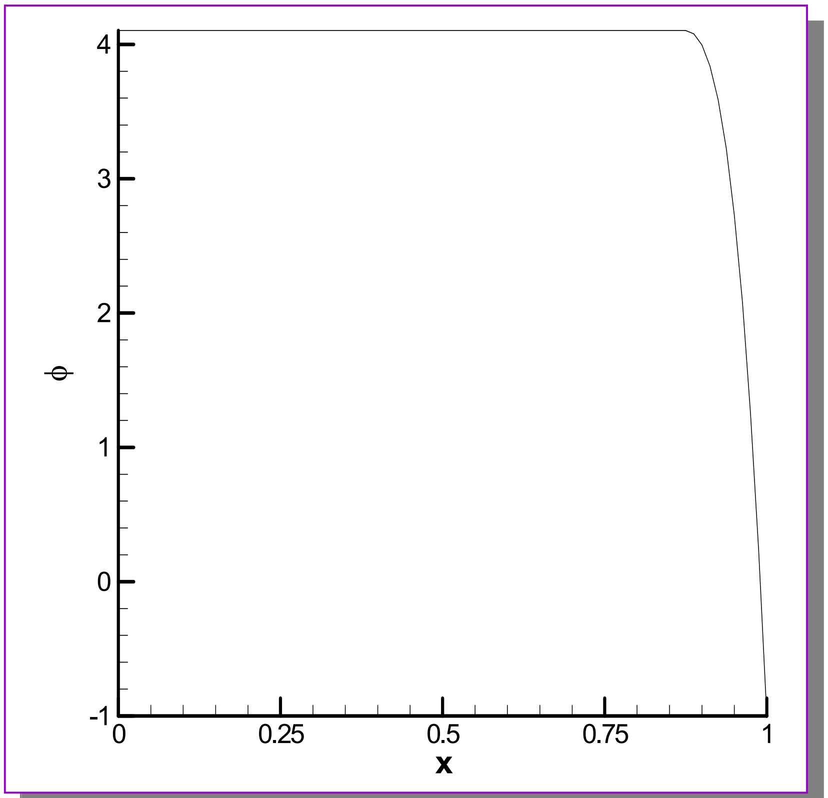


Charge Density Comparison

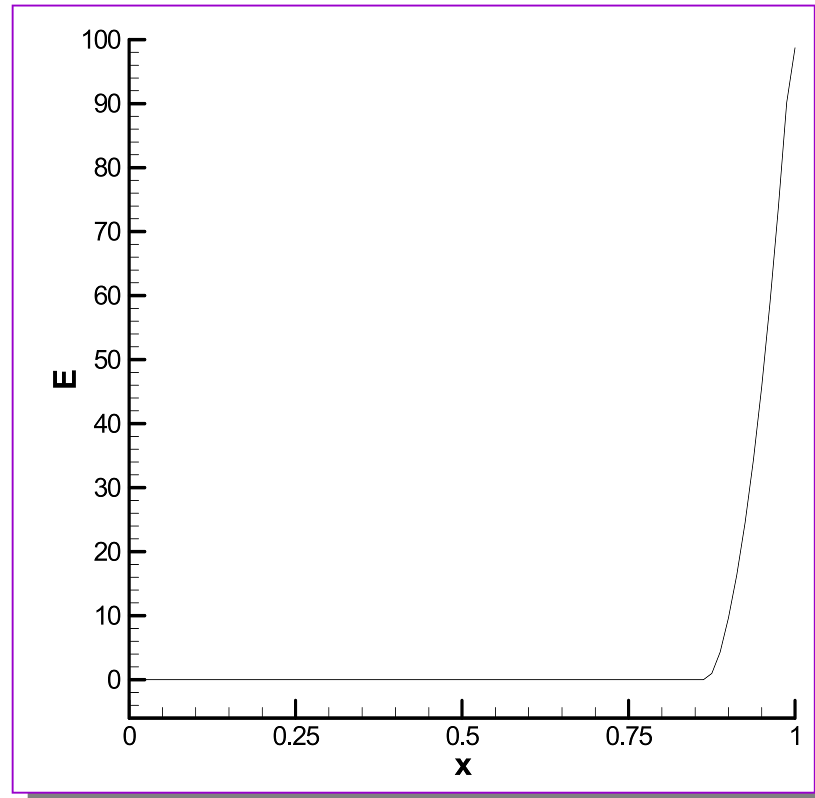


Neutral Number Density

Steady State Solutions
(Potential and Electric field)



Potential Distribution



Electric Field

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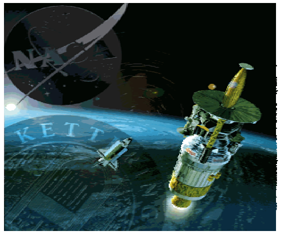
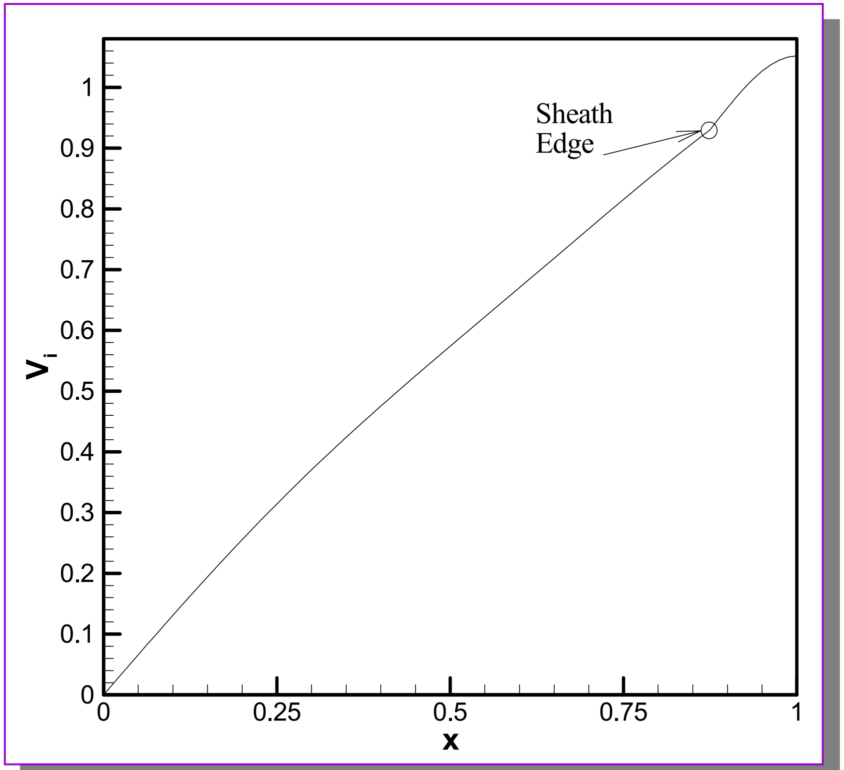
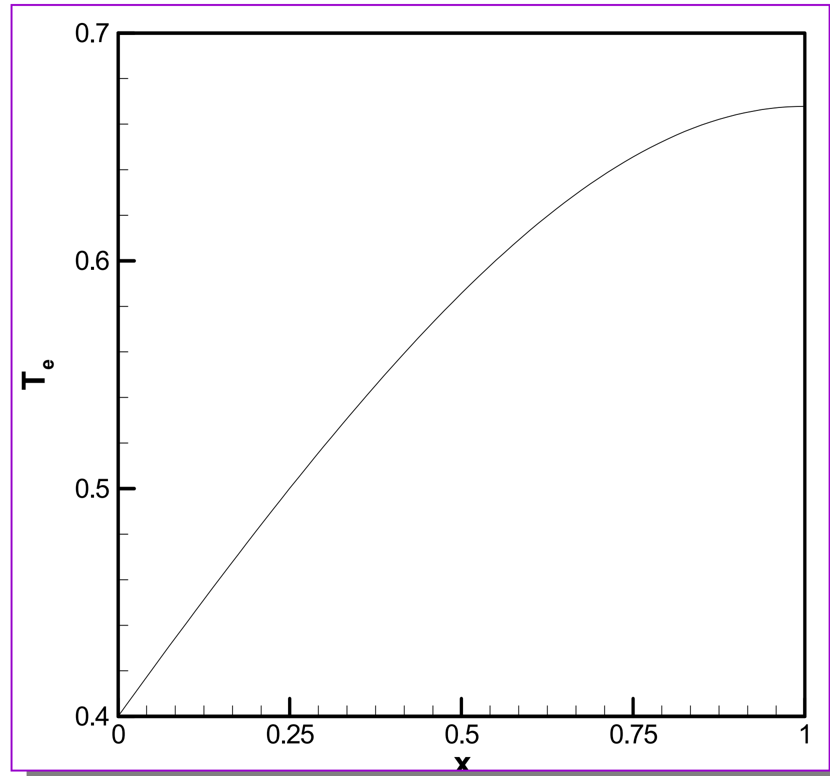


Steady State Solutions

(Ion Velocity and Electron Temperature)



Ion Velocity



Electron Temperature

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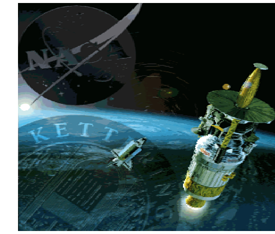
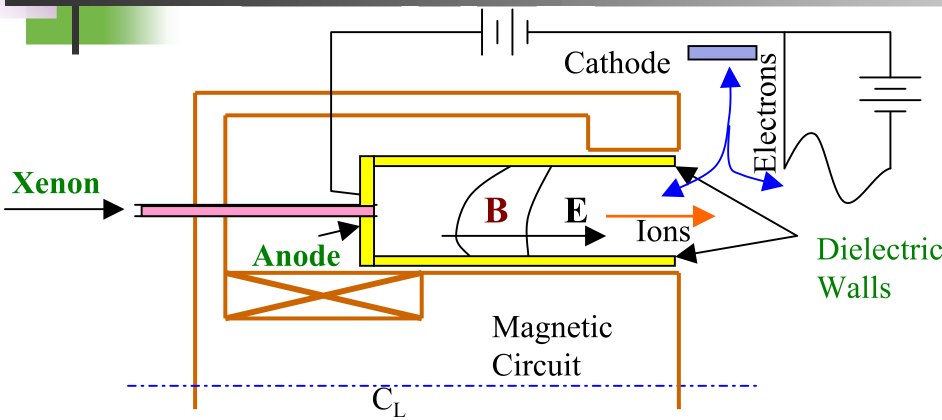
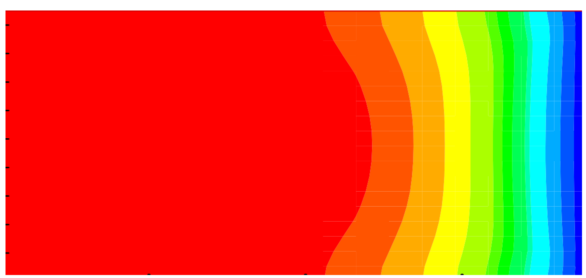


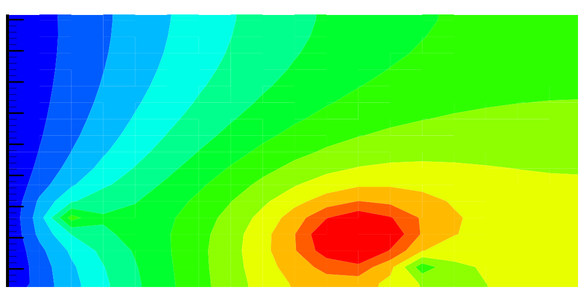
FE Solution for a Hall Thruster

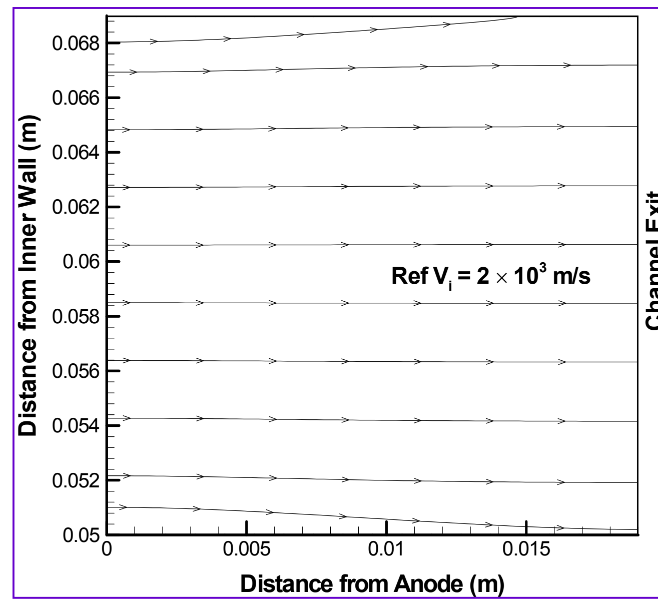


Potential Field



Electron Temperature





Directed Ion Trajectories

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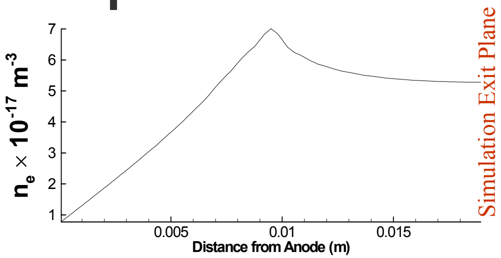
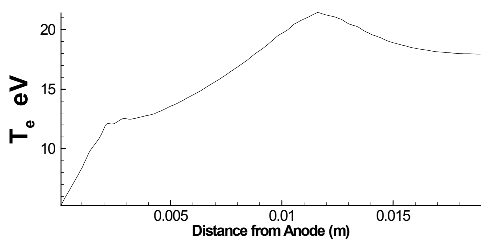
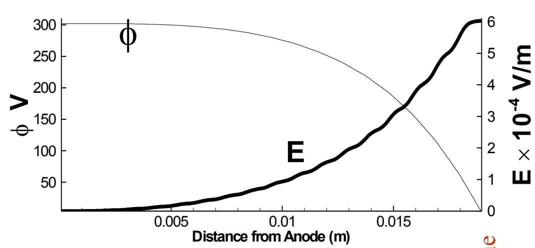
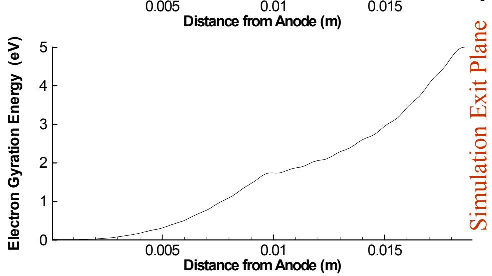
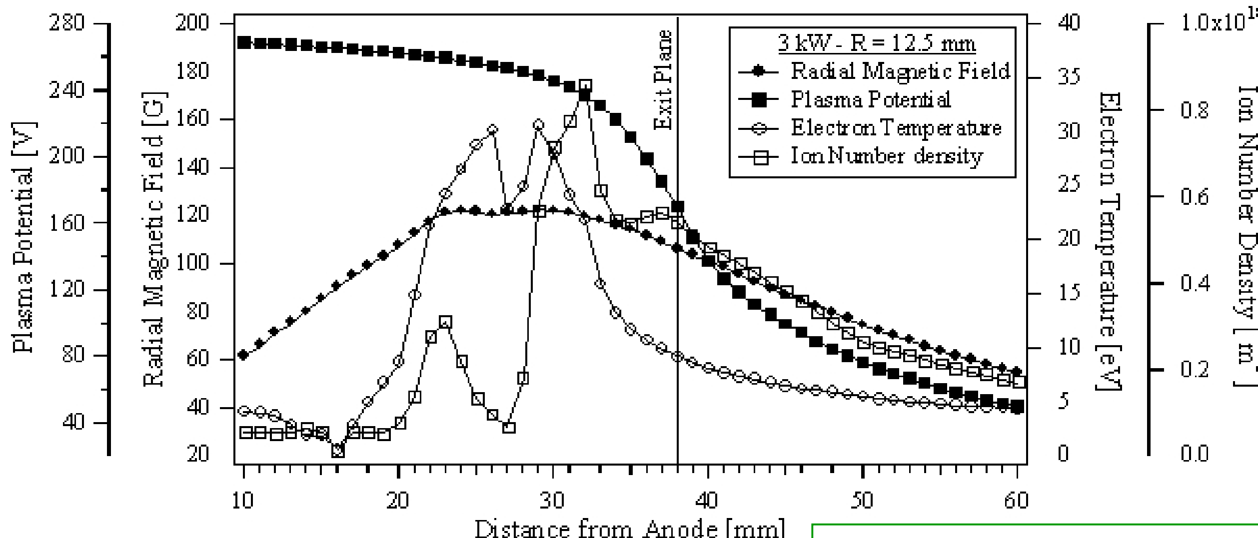
FE Solution Details



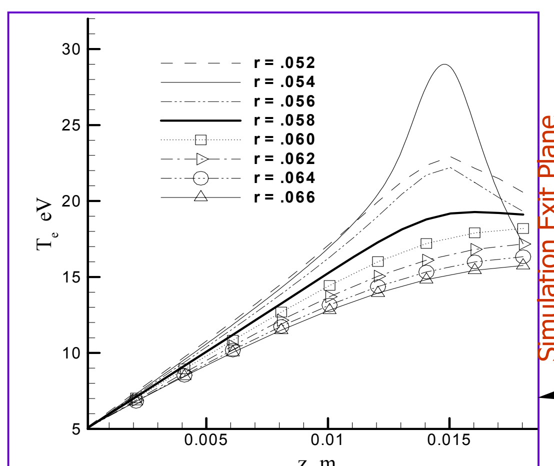








Experimental Data from Haas (PhD, 2001, U. Mich.)



Numerical Results

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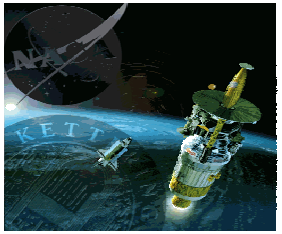

Conclusions

- Finite element code is developed and applied to modeling collisional sheath and bulk plasma.
- Theoretical development for a modified Bohm criteria documented.
- Simulation performed for a single temperature macroscopic partially ionized gas model shows reasonable agreement with recent experiments.
- Understanding the collision effects, geometric and magnetic shape effects inside sheath and plasma-wall interaction will be critical for improved electromagnetic flow control.

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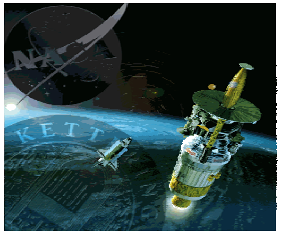
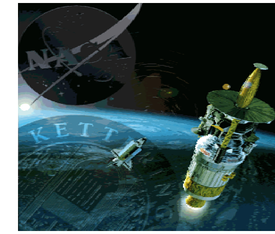


What's Achieved

- Numerical Investigation of Partially Ionized Macroscopic Flow utilizing a new Sub-Grid eMbedded Finite Element Code.
- Inclusion of Ionization and Recombination Effects inside the Sheath-Bulk Plasma.
- Calculation of Electron, Ion and Neutral Number Density Distributions.
- Prediction of Electron and Ion Velocities.
- Calculation of Electron Temperature Distribution.
- Determination of Potential and Electric Field Distribution.



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