COMMENTS

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Comment on "Stationary equilibria of self-gravitating quasineutral dusty plasmas" [Phys. Plasmas 8, 4740 (2001)]

B. P. Pandey and Subrata Roy
Computational Plasma Dynamics Laboratory, Kettering University, Flint, Michigan 48504

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It is pointed out that the recently published study on the stationary equilibria of a self-gravitating quasineutral dusty plasma is not correct. The claim of Rao *et al.* that a "closed form equation for the dust flow speed" [their Eq. (19)] is derived is misleading since a term proportional to the electrostatic potential ϕ_0 has erroneously been left out. Further, the claim of Rao *et al.* that the singularites displayed by their Eqs. (19) and (22) at the characteristic speed are due to the inhomogeneity of the self-gravitating potential is devoid of any mathematical merit or physical reasoning. [DOI: 10.1063/1.1517050]

The equilibrium properties of a self-gravitating medium differ from the equilibrium properties of a nongravitating plasma medium. Whereas, owing to the existence of oppositely charged particles, the zeroth-order electrostatic field can be assumed absent in a plasma medium, the same is not valid for a gravitating medium. Such circumstances lead to two different approaches adopted to study the linear waves and instabilities in these two different media. The plasma medium can be idealized as homogeneous and uniform, which allows for a normal mode analysis of the fluctuations. However, for a self-gravitating medium, one needs to solve an eigenvalue equation with proper boundary conditions. By assuming the zeroth-order gravitational field equal to zero, the gravitational instability was first studied by Jeans,[1] and such an approach has been termed the "Jeans swindle." It must be added here that in many cases, this improper approach to studying the self-gravitating problem gives reasonably good results.[2,3] Therefore, it is important to examine the equilibrium state of a self-gravitating (charged or neutral) system. Recently, Rao *et al.*[4] have attempted to study the stationary equilibrium of a self-gravitating, quasineutral dusty plasma. However, as we shall see, their main equation for that purpose is incorrect. Further, the authors' claim about the nature of singularity is erroneous.

First let us follow Rao *et al.*[4] and critically reexamine their derivation, based upon which they derive "closed form equation for the dust flow speed"—Eq. (19). We start with their quasineutrality condition [Eq. (15)],[4]

$$q_{d0}n_{d0}+e(n_{i0}-n_{e0})=0, \tag{1}$$

where q_{d0} is the charge on the dust grain, e is electronic charge, and n_{e0}, n_{i0}, n_{d0} are the electron, ion, and dust number densities. Following Ref. 4, we shall assume that electron and ion number densities follow Boltzmannian distribution [Eqs. (5) and (6) in Ref. 4],

$$\begin{aligned} n_{e0}&=N_e \exp\left(\frac{e\phi_0}{T_e},\right)\\ n_{i0}&=N_i \exp\left(\frac{-e\phi_0}{T_i},\right) \end{aligned} \tag{2}$$

where $N_{e,i}$ are the number densities of electrons and ions when $\phi_0=0$ and $T_{e,i}$ are electron and ion temperatures. Operating with ∇^2 on Eq. (1), one gets

$$\begin{aligned} q_{d0}\nabla^2 n_{d0}=&\frac{e^2N_i}{T_i}\exp\left(\frac{-e\phi_0}{T_i}\right)\nabla^2\phi_0\\ &+\frac{e^2N_e}{T_e}\exp\left(\frac{e\phi_0}{T_e}\right)\nabla^2\phi_0+\left[\frac{e^3N_e}{T_e^2}\exp\left(\frac{e\phi_0}{T_e}\right)\right.\\ &\left.-\frac{e^3N_i}{T_e^2}\exp\left(\frac{-e\phi_0}{T_i}\right)\right]\left(\frac{\partial\phi_0}{\partial r}\right)^2. \end{aligned} \tag{3}$$

Defining some quantity λ_D [which still contains potential and thus is not the usual Debye length, contrary to the claim in Ref. 4],

$$\lambda_D^{-2}(\phi_0)=\frac{e^2N_i}{\epsilon_0 T_i}\exp\left(\frac{-e\phi_0}{T_i}\right)+\frac{e^2N_e}{\epsilon_0 T_e}\exp\left(\frac{e\phi_0}{T_e}\right), \tag{4}$$

Eq. (3) can be written as [Eq. (17) in Ref. 4]

$$\begin{aligned} q_{d0}\nabla^2 n_{d0}=&\frac{\epsilon_0}{\lambda_D(\phi_0)}\nabla^2\phi_0+\left[\frac{e^3N_e}{T_e^2}\exp\left(\frac{e\phi_0}{T_e}\right)\right.\\ &\left.-\frac{e^3N_i}{T_e^2}\exp\left(\frac{-e\phi_0}{T_i}\right)\right]\left(\frac{\partial\phi_0}{\partial r}\right)^2. \end{aligned} \tag{5}$$

Now comes the crucial step. ‘‘In order to obtain a *closed form equation for the dust flow speed, we neglect the nonlinear term* in Eq. (17) [of Ref. 4]’’ or our Eq. (5). Then

$$\nabla^2 n_{d0} = \frac{\epsilon_0}{\lambda_D^2 q_{d0}} \nabla^2 \phi_0 . \tag{6}$$

This approximation is essentially the same as requiring $e\phi_0 \ll T_e,\ T_i$.[4] As Rao *et al.*[4] have used this equation in Bernouli's equation (13) to get a *closed form, Eq. (19) for the dust flow speed*, one may first work out the consequence of Eq. (6) on the gravitational potential before eliminating $\nabla^2\psi_0$ in favor of ω_J^2 in Bernouli's equation [Eq. (12) in Ref. 4]. Equation (6) can be written as

$$\nabla \left(n_{d0} - \frac{\epsilon_0}{\lambda_D^2 q_{d0}} \phi_0 \right) = C_1 , \tag{7}$$

where C_1 is a constant. Equations (6) and (7) display a direct relation betweeen n_{d0} and ϕ_0 and, thus, gravitational potential gets intimately linked to the electrostatic potential. In order to see it clearly, let us choose $C_1 = 0$ and integrate it once more. Then

$$n_{d0} - \frac{\epsilon_0}{\lambda_D^2 q_{d0}} \phi_0 = C \tag{8}$$

and in $\phi_0 \to 0$ limit $C \equiv n_{d0} = (e/q_{d0})(N_e - N_i)$ [from Eq. (1)] and we are led to the following relation between dust density and potential:

$$q_{d0} n_{d0} + e(N_i - N_e) - \left(\frac{\epsilon_0}{\lambda_D^2} \right) \phi_0 = 0. \tag{9}$$

One could have obtained the same equation (9) by assuming $e\phi_0 \ll T_e,\ T_i$ and expanding (1), but then one would have missed the redundant nonlinear (in ϕ_0) Eq. (17) in Ref. 4 [Eq. (5) in the present text] which remains unused. Due to direct relation between n_{d0} and ϕ_0, Poisson's equation. for gravitational potential ψ_0 gets modified,

$$\nabla^2 \psi_0 = 4\pi G m_d \left[\frac{e(N_e - N_i)}{q_{d0}} + \left(\frac{\epsilon_0}{q_{d0}\lambda_D^2} \right) \phi_0 \right]. \tag{10}$$

Thus, operating on Bernouli's equation with ∇^2 gives

$$\frac{1}{2}\nabla^2 u_{d0}^2 + \frac{q_{d0}}{m_d}\nabla^2\phi_0 + v_{Td}^2 \nabla^2 \ln n_{d0} + \omega_{Jd}^2 + \left(\frac{4\pi G m_d \epsilon_0}{q_{d0}\lambda_D^2} \right) \phi_0 = 0. \tag{11}$$

Equation (11) has one more term than Eq. (13) of Ref. 4. The last term, which reflects a coupling between the electrostatic and the gravitational forces (a fact noted by Pandey *et al.*[5]), is absent in Ref. 4. As a result, Eq. (19) and the subsequent discussion of the result of Rao *et al.*[4] is erroneous. Rao *et al.* may well argue that Jeans frequency ω_J is defined in terms of n_{d0} and hence it contains all that we are saying here. But then Eq. (19) of Rao *et al.*[4] is not a closed form equation in terms of dust flow speed and a term directly proportional to ϕ_0 is present in it. Therefore, either the claim that Eq. (19) is a closed form equation for the dust flow speed is misleading or the equation is plainly wrong.

Next, let us come to the singularity of Eqs. (19) and (22), which is claimed to be a consequence of the inhomogeneous equilibrium self-gravitational potential. If the claim of the authors of Ref. 4 is correct, then such a singularity should disappear from their Eqs. (19) and (22) when self-gravity is absent. Let us assume that there is no self-gravity. Then, Eq. (19) of Rao *et al.*[4] becomes

$$u_{d0}(u_{d0}^2 - c_{da}^2 - v_{Td}^2)\frac{\partial^2 u_{d0}}{\partial r^2} + (u_{d0}^2 + 2c_{da}^2 + v_{Td}^2)\left(\frac{\partial u_{d0}}{\partial r}\right)^2 + \frac{\nu u_{d0}}{r}(u_{d0}^2 + c_{da}^2 - v_{Td}^2)\left(\frac{\partial u_{d0}}{\partial r}\right) + \left(\frac{\nu c_{da}^2}{r^2} - \frac{\nu(\nu-1)v_{Td}^2}{r^2} \right) u_{d0}^2 = 0. \tag{12}$$

This equation still displays a singularity at $u_{d0}^2 - c_{da}^2 - v_{Td}^2 = 0$. Evidently, self-gravity has nothing to do with the singularity. Therefore, the claim of Rao *et al.* that ‘‘singularity is a consequence of the inhomogeneous equilibrium self-gravitational potential which manifests itself in the governing equation through Jeans frequency’’ is untrue.

Let us see how Rao *et al.* managed to get such a singularity and what the origin of such a singularity is. In order to understand the physical origin of singularity, without loss of generality, we shall assume Cartesian one-dimensional geometry and assume cold dust. Then in the presence of flow, the dust momentum equation gives

$$\frac{u_{d0}^2}{2} + \frac{q_d \phi_0}{m_d} = C_1 . \tag{13}$$

Now making use of $n_{d0}u_{d0} = C_2$, one can write

$$n_{d0} = C_2 \left[2\left(C_1 - \frac{q_d\phi_0}{m_d} \right) \right]^{-1/2} . \tag{14}$$

The boundary condition will require $C_2 = n_{d00}u_{d00}$ and $C_1 = m_d u_{d00}^2/2$ as we must have $\phi_0(\infty) = 0$, $n_{d0}(\infty) = n_{d00}$ and $v_{d0}(\infty) = v_{d00}$. Poisson's equation becomes

$$\frac{d^2\phi_0}{dx^2} = -\frac{1}{\epsilon_0}\left(eN_e\left(\exp\left(\frac{-e\phi_0}{T_i} \right) - \exp\left(\frac{e\phi_0}{T_e} \right) \right) - \frac{q_d n_{d00}}{\left[\left(1 - \frac{2q_d\phi_0}{m_d u_{d00}^2} \right) \right]^{1/2}} \right). \tag{15}$$

Expanding the above-given expression around $\phi_0 = 0$, to the lowest order, one recovers quasineutrality [Eq. (1)] and in the next order

$$\frac{d^2\phi_0}{dx^2} = \left(\frac{1}{\lambda_D^2} - \frac{\omega_{pd}^2}{v_{d0}^2} \right)\phi_0 . \tag{16}$$

The role of the second term on the right-hand side varies. Whereas the first term represents the usual Debye shielding, the second term represents ''anti-shielding''[6] due to the flow of particles. Equation (11) can be written as

$$\frac{d^2\phi_0}{dx^2} - \frac{\phi_0}{\chi^2} = 0, \tag{17}$$

where

$$\chi^2 = \frac{\lambda_D^2}{1 - \frac{c_{da}^2}{v_{d0}^2}}, \quad c_{da} = \lambda_D \omega_{pd}.$$

The solution of Eq. (12) is

$$\phi_0 = A \exp\left(\frac{-x}{\chi}\right). \tag{18}$$

When $v_{d00} = c_{da}$, i.e., $\chi^2 \to \infty$, $\phi_0 \to A$, where A is determined from the boundary condition imposed on the potential. In a bounded plasma, generally A is equated to the wall potential. Therefore, at $v_{d0} = c_{da}$, i.e., when shielding is exactly canceled by the ''anti-shielding,'' the plasma structure consists of the thin non-neutral sheaths tied to the boundary and a quasineutral region (the presheath) tied to the bulk of the channel. The condition for the transition between the presheath/sheath region is unique and consists in plasma flow being sonic there. The ''local singularity'' of Ref. 4 is just a manifestation of this transition at Bohm velocity and has nothing to do with ''inhomogeneous self-gravitational potential.''

The condition for the removal of such a singularity in a two component plasma has been extensively discussed by Freedman and Levi.[7] Rao *et al.* can benefit from it and generalize it to three-component plasma without any difficulty.

The singularity in their equation (22) for a neutral fluid also survives the ''zero self-gravity'' test, i.e., inhomogeneity of self-gravity has nothing to do with the singularity. This singularity is well known in hydrodynamics[8] and occurs when a transition from subsonic to supersonic flow takes place.

To summarize, the paper by Rao *et al.*[4] on the stationary equilibrium of a self-gravitating quasineutral dusty plasma is neither algebraically correct nor provides a physically correct interpretation of the singularity displayed by the equation. Erroneously, singularity has been attributed to the inhomogeneity of self-gravitational potential. The singularity displayed by their equation (19) (for a dusty plasma) or Eq. (22) for a neutral fluid survives in the absence of self-gravity. However, when self-gravity is present and electric field is absent, one can see from their Eq. (19) that for a cold dust, no singularity exists. Similar comments are valid for the neutral fluid equation (22).

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